Communications (Telemetry & Command)

MAE 4160, 4161, 5160

V. Hunter Adams, PhD

Today’s topics:

- Link budget equation
- Modulations
- Bit error rate
- Shannon Limit
- Coding techniques
- Antennas
- Examples

A simple communications architecture

More complex communications architecture

The Link Budget Equation

What it tells us: The ratio of the received power from our spacecraft to the power of noise in our system. In other words, the *signal-to-noise ratio*.

$$SNR = \frac{P_R}{P_N}$$

We will consider the numerator and denominator separately, starting with the numerator. What terms to you expect will appear in the expression for the received signal power?

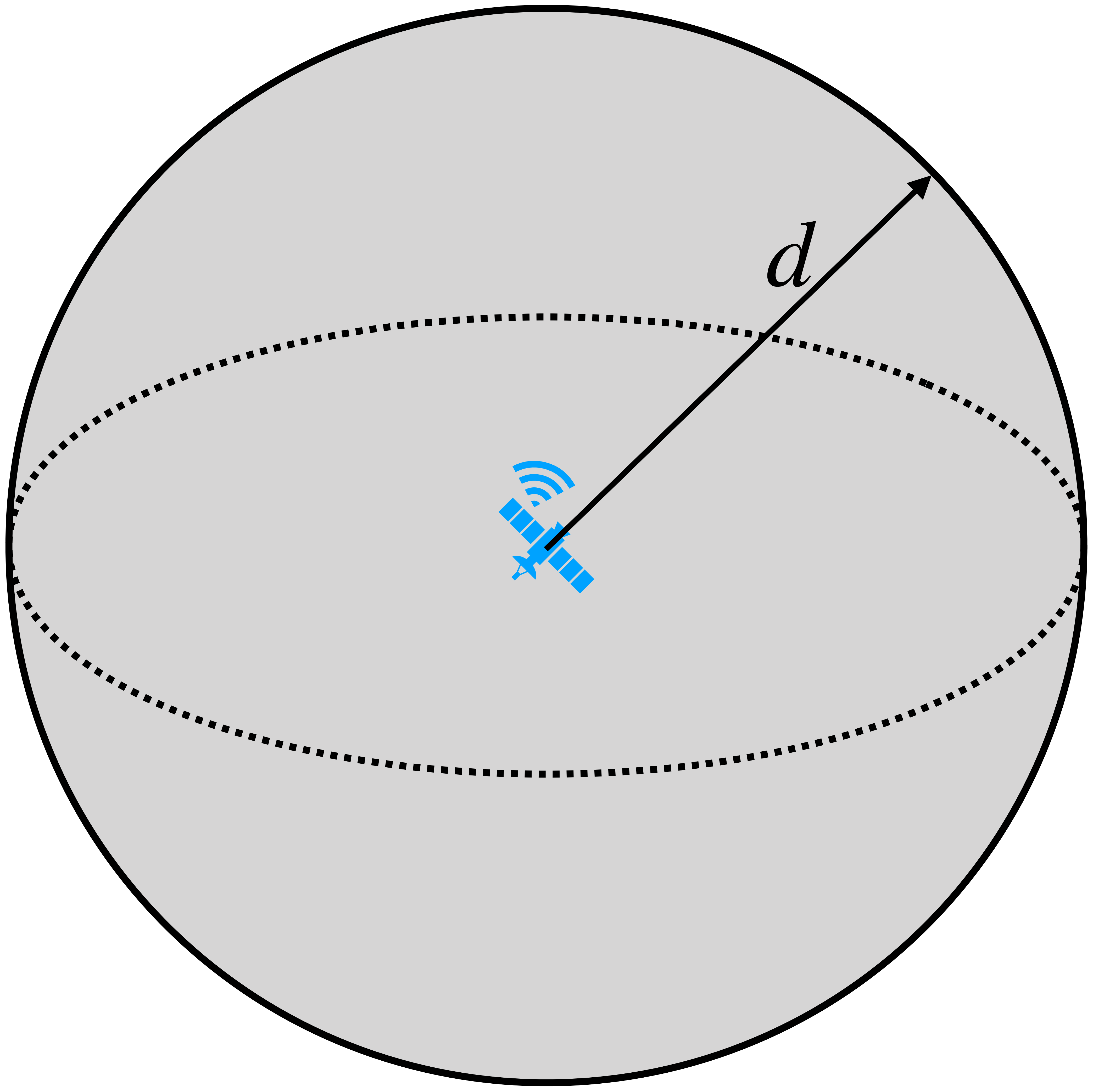
The Link Budget Equation: Received Power

Consider the case of two antennas (TX and RX) in free space separated by a distance d.

Assume P_T total Watts of power is delivered to the transmit antenna which (for the moment) is assumed to be omnidirectional and lossless.

Power density decreases with distance.

$$P_R = \frac{P_T}{4\pi d^2}$$



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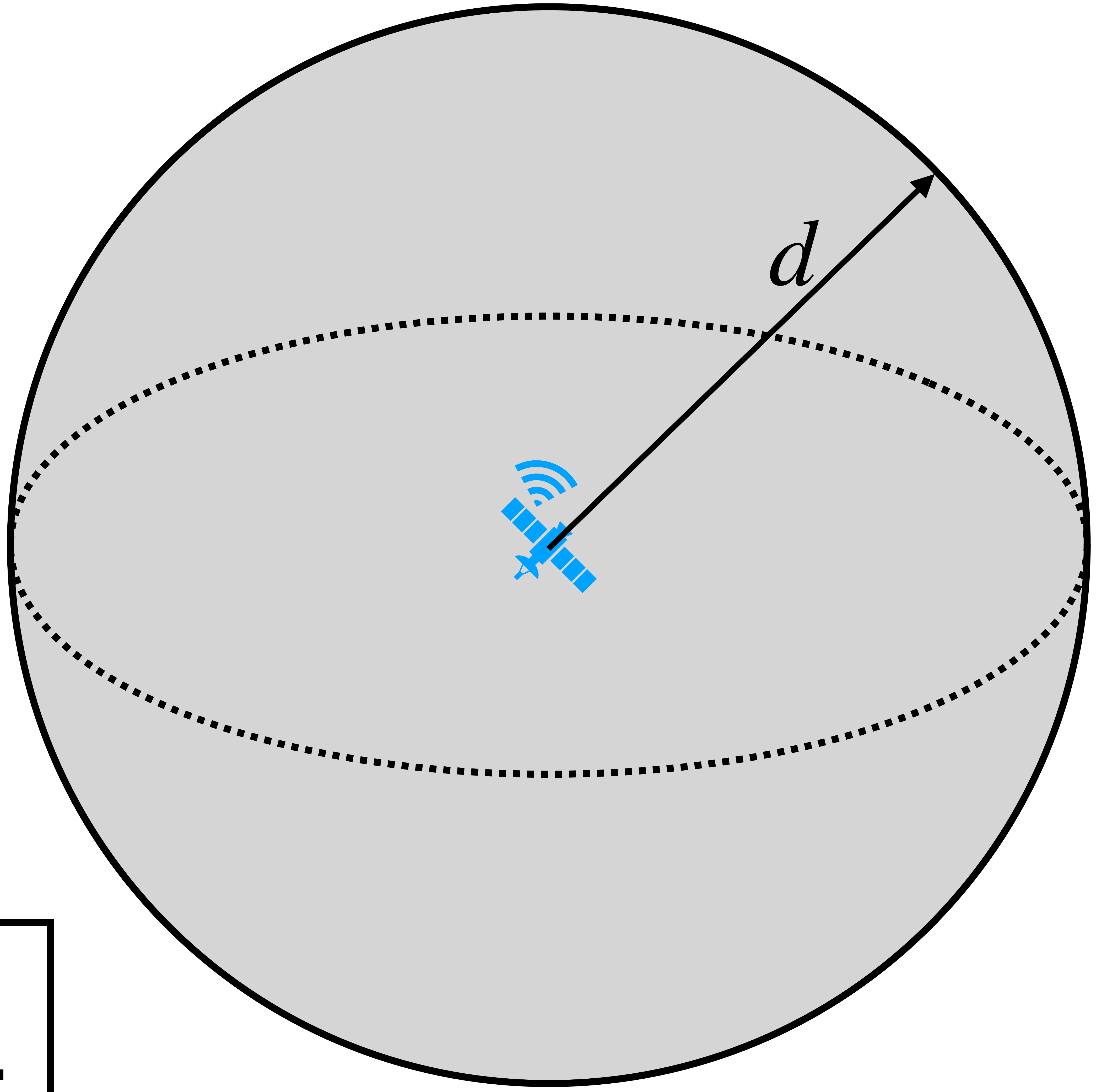
Now we'll add details.



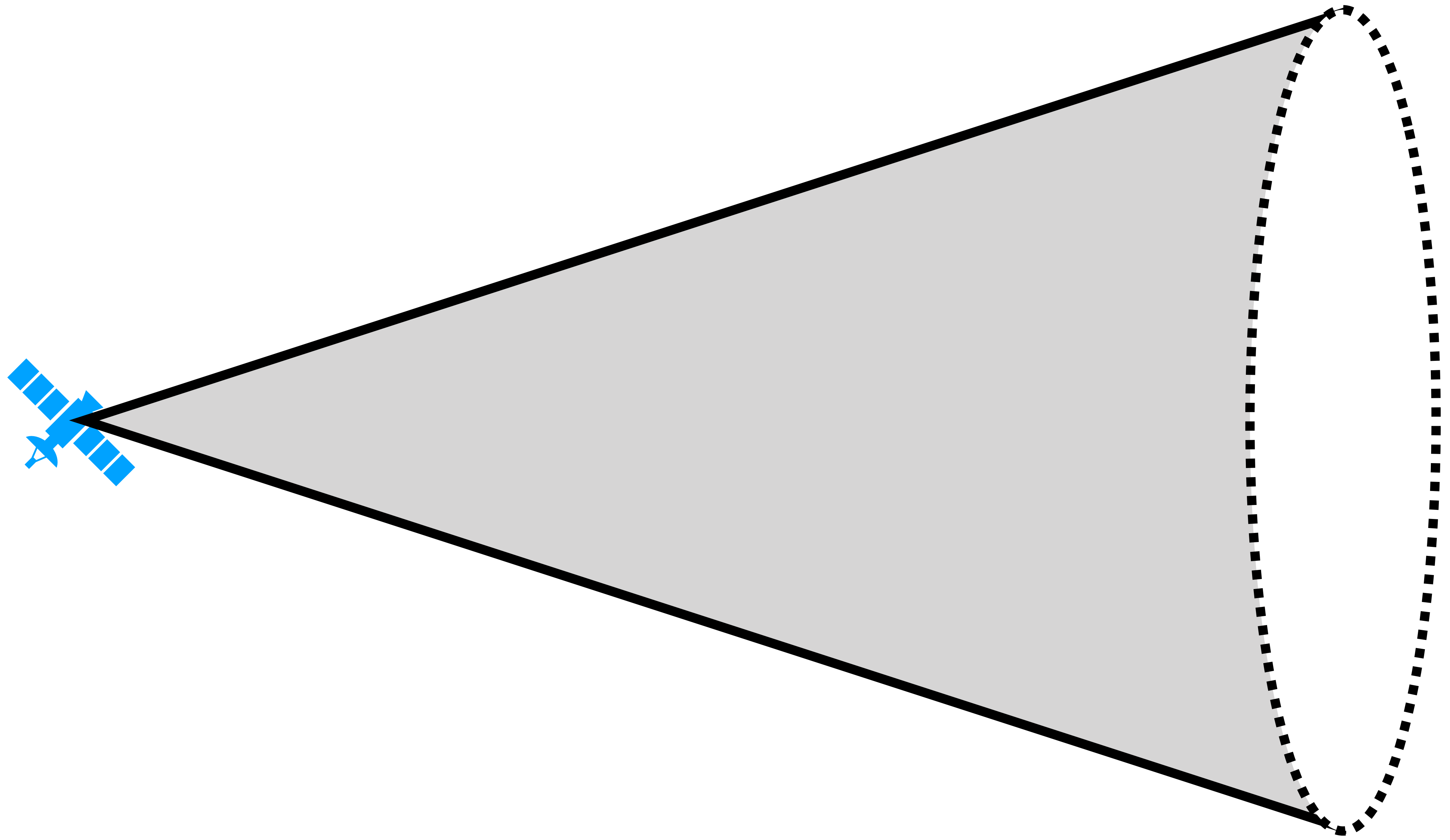
The Link Budget Equation: Received Power

1. Distance loss

$$P_R = \frac{P_T}{4\pi d^2}$$

2. Transmit antenna gain/directionality

$$P_R = \frac{P_T}{4\pi d^2} G_T$$



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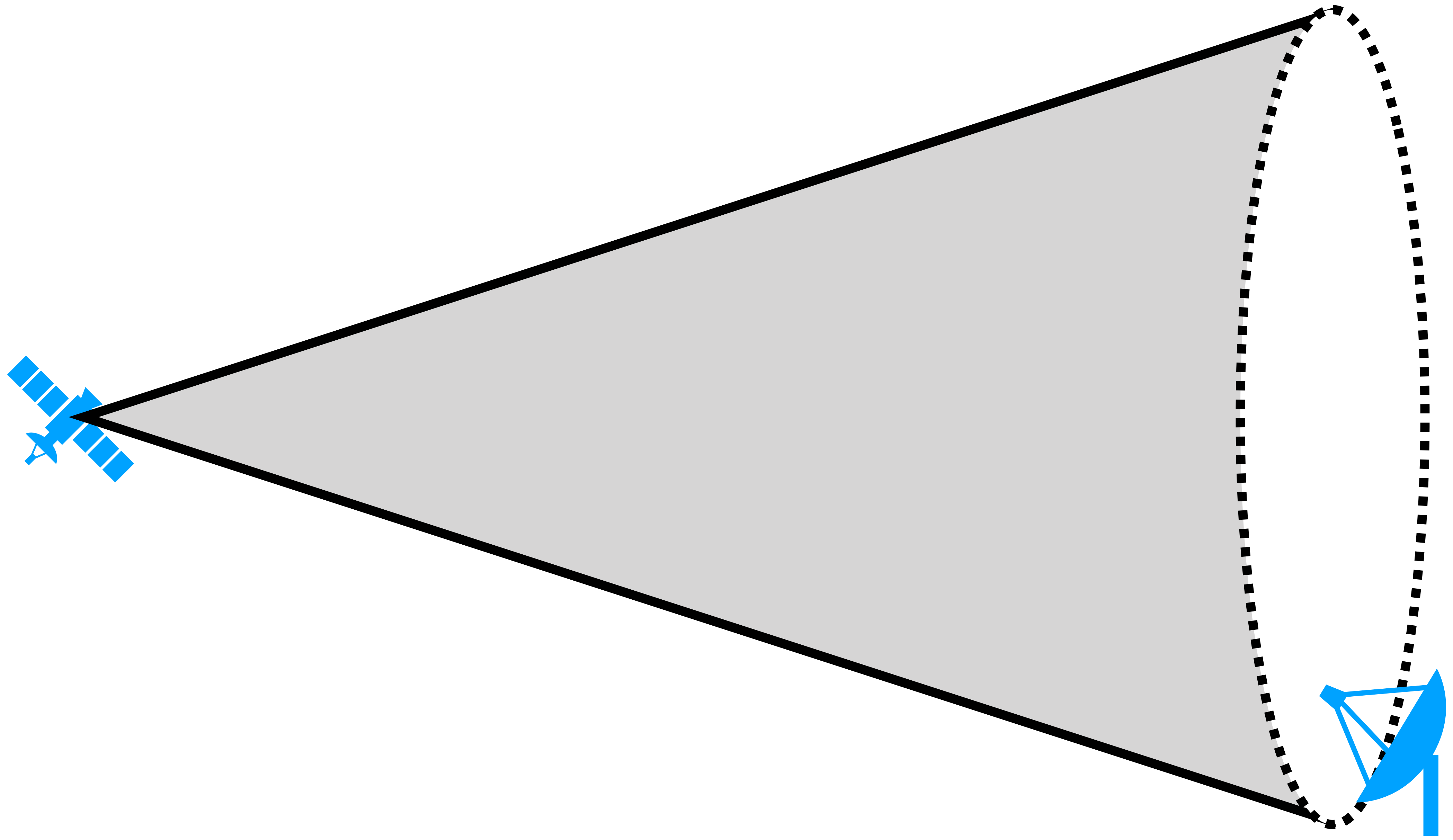
$$P_R = \frac{P_T}{4\pi d^2} G_T A_{ER}$$



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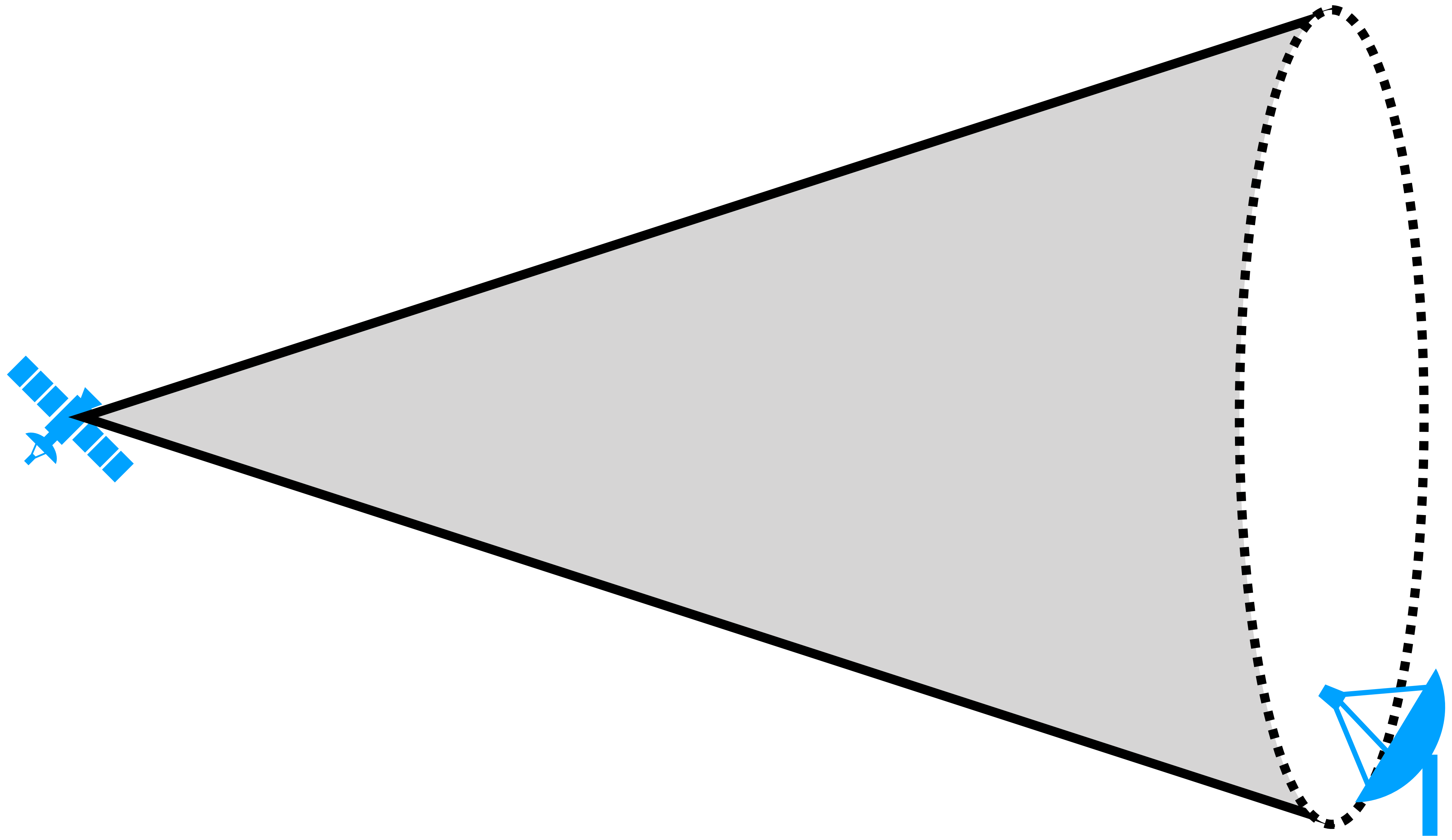
4. Rewrite equation for aperture

$$P_R = \frac{P_T}{4\pi d^2} G_T \cdot \frac{\lambda^2}{4\pi} G_R$$

$$= \frac{P_T G_T G_R \lambda^2}{(4\pi d)^2}$$



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This is the ***Friis Transmission Formula.*** Often, this is written in units of decibels:

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This is the ***Friis Transmission Formula.*** Often, this is written in units of decibels:

$$10\log_{10}(P_R) = 10\log_{10}\left(\frac{P_T G_T G_R \lambda^2}{(4\pi d)^2}\right)$$

$$\boxed{[P_R]_{db} = [P_T]_{db} + [G_T]_{db} + [G_R]_{db} + 10\log_{10}\left[\left(\frac{\lambda}{4\pi d}\right)^2\right]}$$

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But our channel is not lossless! We have attenuation from the atmosphere and from circuit losses.

The Link Budget Equation: Received Power

This is the **lossless** equation that we just derived. We will augment this equation to account for **atmospheric** and **circuit** attenuation.

$$P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi d)^2}$$

Remember what we're doing?

$$SNR = \frac{P_R}{P_N}$$

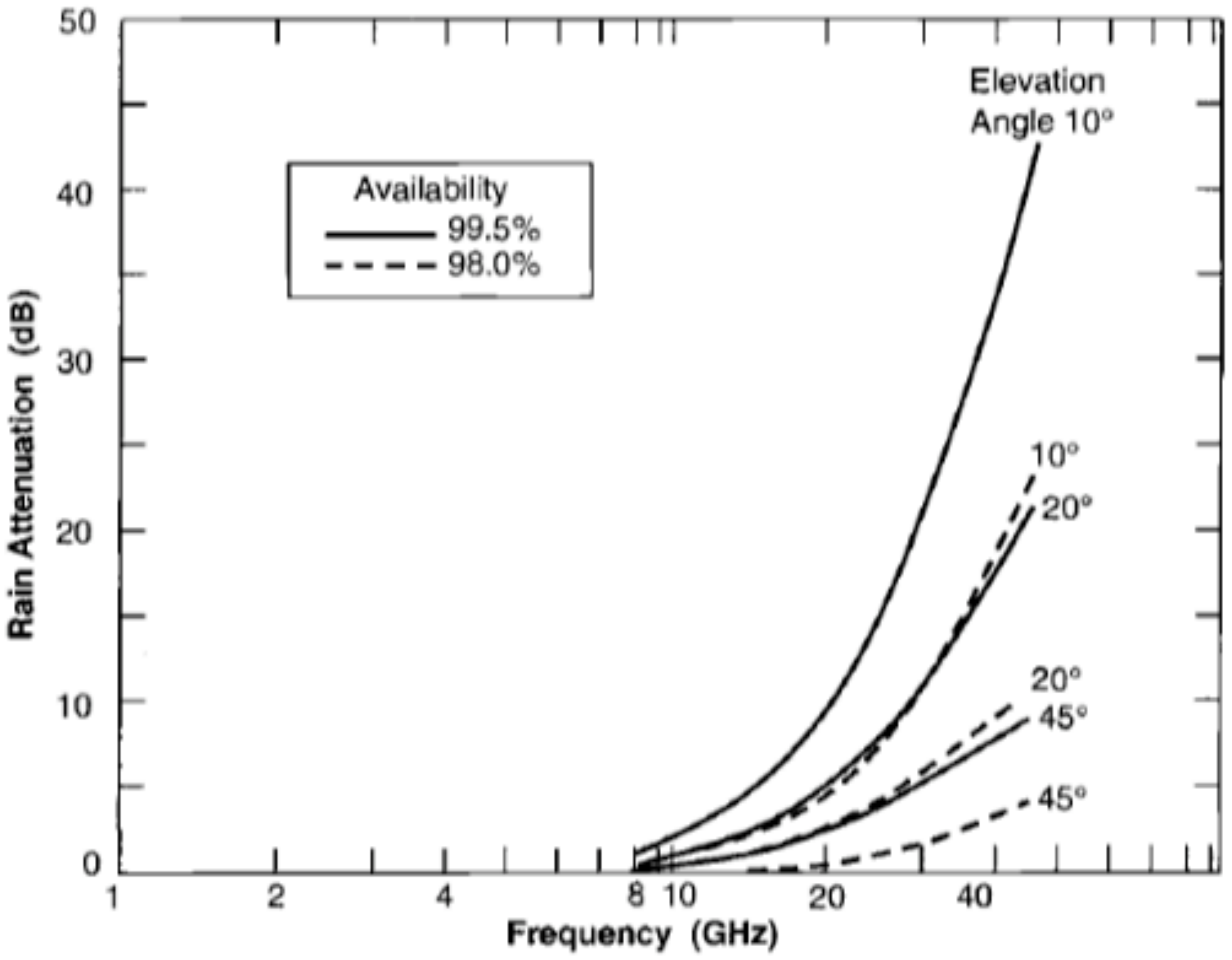
The Link Budget Equation: Received Power

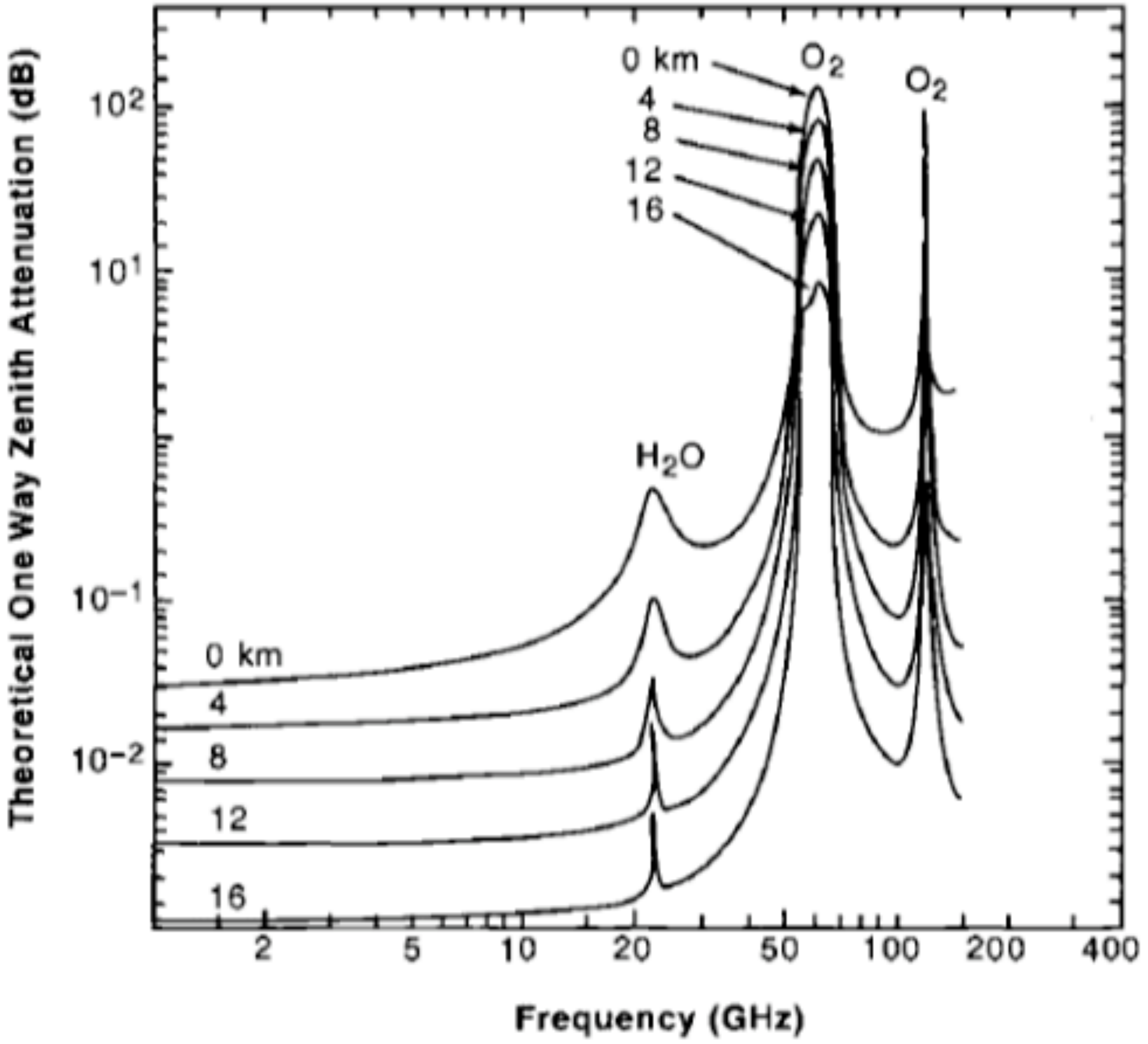
1. Atmospheric attenuation

Transmission through the atmosphere attenuates the signal by some scale factor L_a. This will be some number between 0 and 1. We can augment our received power equation to include this term:

$$P_R = \frac{P_T G_T G_R L_a \lambda^2}{(4\pi d)^2}$$

The amount of attenuation depends on one's choice of frequency.





The Link Budget Equation: Received Power

1. Circuit attenuation

Our signal will also be attenuated by our hardware (coaxial cables, connectors, etc.). Transmission through this hardware will attenuate the signal by some scale factor L_l. Augment our equation once more:

$$P_R = \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2}$$

The Link Budget Equation: Received Power

We're almost there. What are the units of the below equation?

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The Link Budget Equation: Received Power

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$$P_R = \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2}$$

Watts. Or Joules/sec. We often want to know the energy per *bit* rather than the energy per *second*. To find the energy per bit (E_b), we simply divide by our data rate R_b (bits/sec):

$$\begin{aligned} E_b &= \frac{P_R}{R_b} \\ &= \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2 R_b} \longrightarrow \textbf{Joules per bit} \end{aligned}$$

The Link Budget Equation: Received Power

We're almost there. What are the units of the below equation?

We have our numerator!

$$SNR = \frac{P_R}{P_N}$$

Now for the denominator . . .

Watts. Or Jou… …e energy per *bit* rather than the… …ergy per bit (E_b), we simply divi…

$$E_b = \frac{P_R}{R_b}$$

$$= \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2 R_b} \longrightarrow \textbf{Joules per bit}$$

The Link Budget Equation: Noise Power

We want to compare the energy contribution from our signal (below) to the energy contribution from noise.

$$E_b = \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2 R_b}$$

The majority of our noise is *thermal*. The spectral noise density for thermal noise is calculated as shown below:

Noise power (W)

Boltzmann constant (J/K)

$$N_0 = \frac{P_N}{B} = K_B T_{sys}$$

Bandwidth (Hz)

System noise temperature (K)

The Link Budget Equation: Noise Power

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Brief aside on system noise temperature . . .

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Bandwidth (Hz)

System noise temperature (K)

System Noise Temperature, T_{sys}

The total noise temperature has contributions from the antenna and the receiver:

$$T_{sys} = T_A + T_R$$

Antenna noise temperature, T_A, gives the noise power seen at the output of the antenna. The receiver noise temperature, T_R, represents noise generated by components inside the receiver. Often, we use the *noise factor* F to represent receiver noise temperature.

$$T_{sys} = T_A + T_0(F - 1)$$

Where $T_0 = 290K$.

System Noise Temperature, T_{sys}

The total noise temperature has contributions from the antenna and

Noise Temperature	Frequency (GHz)					
	Downlink			Crosslink	Uplink	
	0.2	2–12	20	60	0.2–20	40
Antenna Noise (K)	150	25	100	20	290	290
Line Loss (dB)	0.5	0.5	0.5	0.5	0.5	0.5
Line Loss Noise (K)	35	35	35	35	35	35
Receiver Noise Figure (dB)	0.5	1.0	3.0	5.0	3.0	4.0
Receiver Noise (K)	36	75	289	627	289	438
System Noise (K)	221	135	424	682	614	763
System Noise (dB-K)	23.4	21.3	26.3	28.3	27.9	28.8

$$T_{sys} = T_A + T_0(F - 1)$$

Where $T_0 = 290K$.

The Link Budget Equation

We are interested in the ratio of signal energy to noise energy. We now have everything we need to compute that.

$$\frac{E_b}{N_0} = \frac{P_T G_T G_R L_a L_l \lambda^2}{\left(4\pi d\right)^2 K_B T_{sys} R_b} \longrightarrow$$ **signal to noise ratio per bit**

$$\left[\frac{E_b}{N_0}\right]_{db} = [P_T]_{db} + [G_T]_{db} + [G_R]_{dB} + [L_a]_{dB} + [L_l]_{dB} + 10\log_{10}\left[\left(\frac{\lambda}{4\pi R}\right)^2\right] - 10\log_{10}\left(K_B T_{sys} R_b\right)$$

$$\frac{P_R}{P_N} = \frac{P_T G_T G_R L_a L_l \lambda^2}{\left(4\pi d\right)^2 K_B T_{sys} B} \longrightarrow$$ **signal to noise ratio per bandwidth**

$$\left[\frac{S}{N}\right]_{db} = [P_T]_{db} + [G_T]_{db} + [G_R]_{dB} + [L_a]_{dB} + [L_l]_{dB} + 10\log_{10}\left[\left(\frac{\lambda}{4\pi R}\right)^2\right] - 10\log_{10}\left(K_B T_{sys} B\right)$$

The Link Budget Equation

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$P_T G_T$: **Equivalent isotropic radiated power (EIRP)**

$\left(\dfrac{\lambda}{4\pi d}\right)^2$: **Free space loss L_s**

$\dfrac{G_R}{T_{sys}}$: **Receiver gain to noise temperature**

$\left[\dfrac{E_b}{N_0}\right]_{db}$ … $\left(K_B T_{sys} R_b\right)$

…idth

P_N … $(4\pi d)^2 K_B T_{sys} B$

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- Shannon Limit
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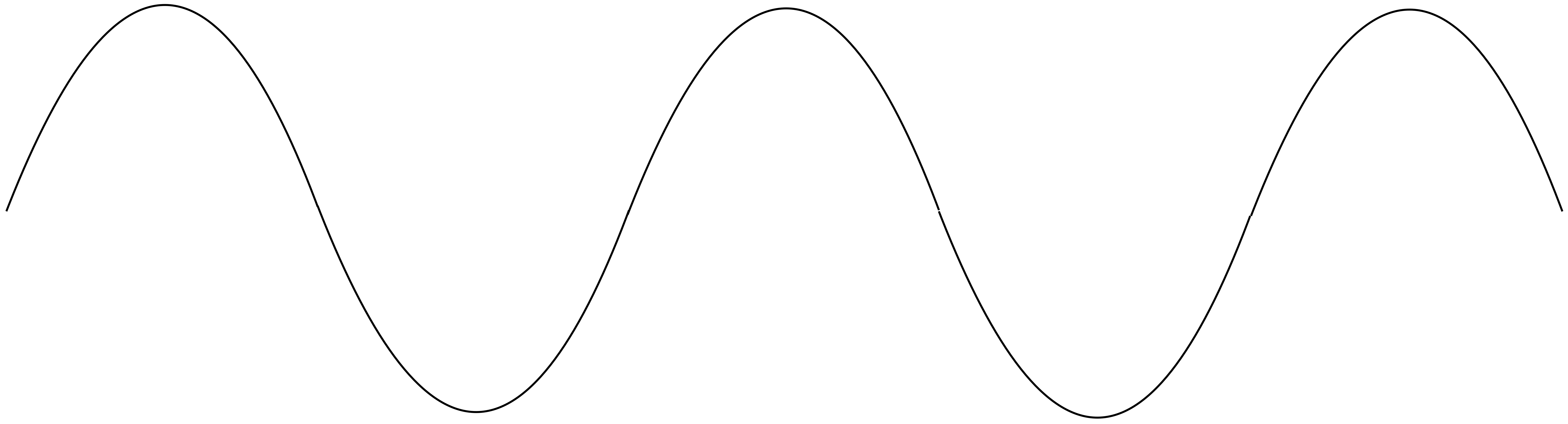
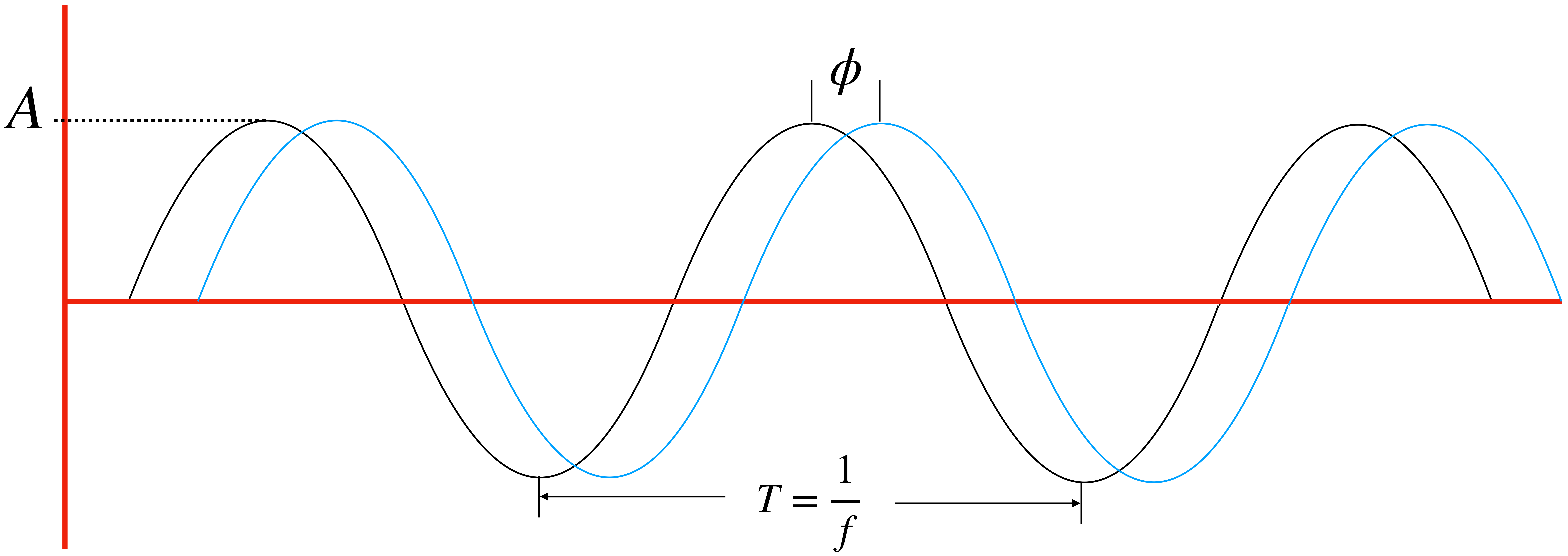
An electromagnetic wave



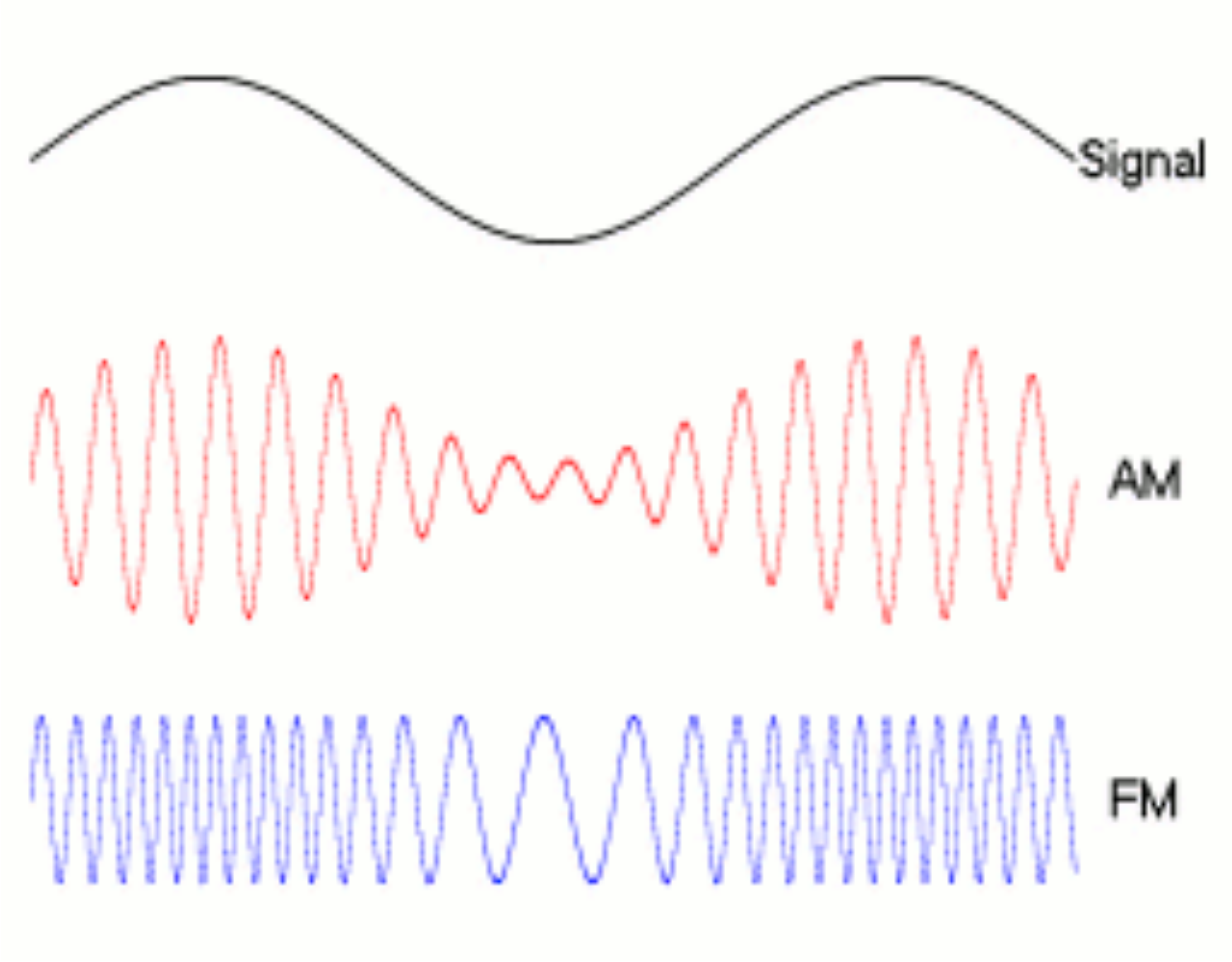
What are our degrees of freedom?

An electromagnetic wave



What are our degrees of freedom?

Analog modulations: AM and FM

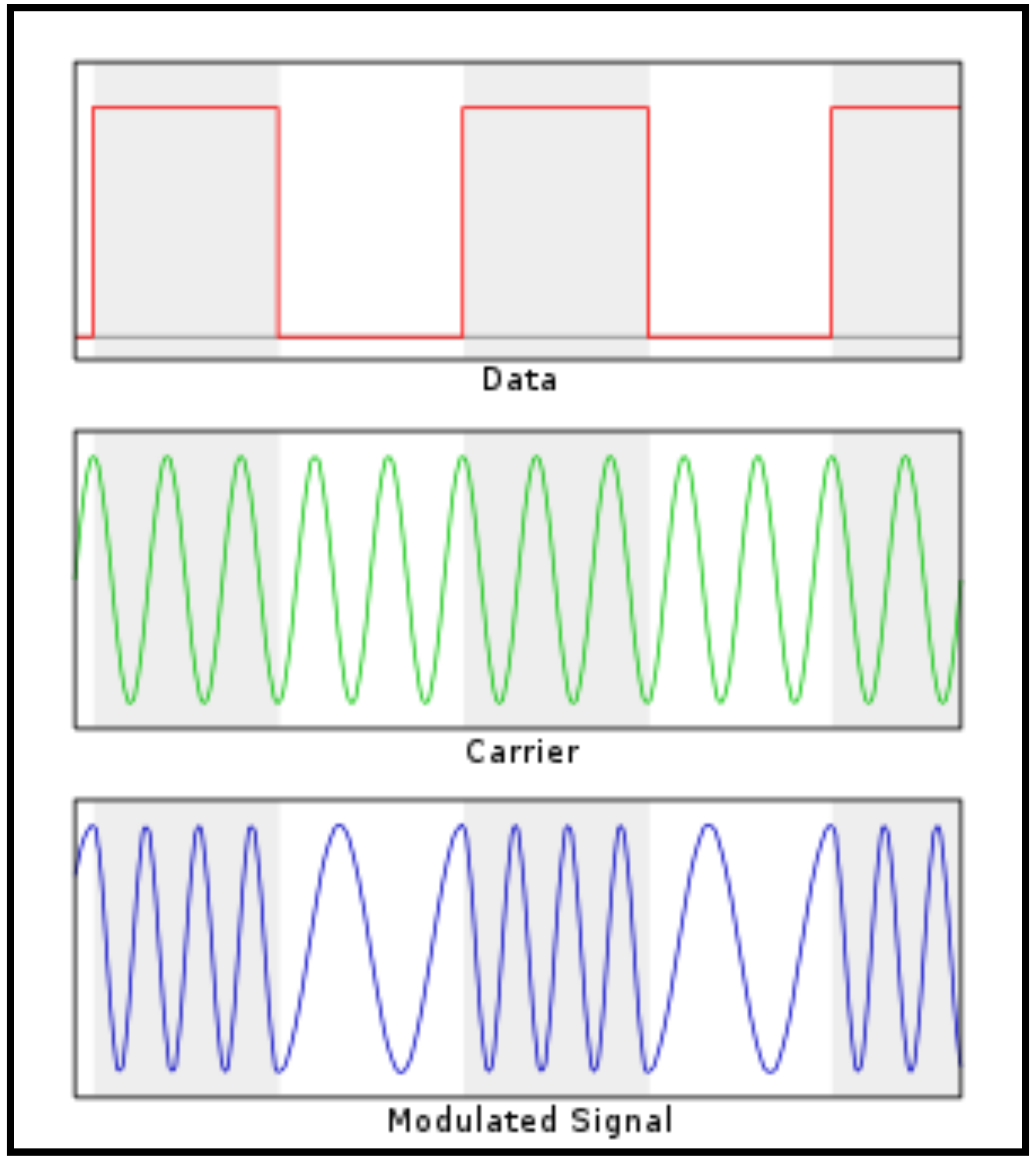


We need modulation because we must carry our low-frequency data on high-frequency waves. Otherwise, we'd require huge antennas, and our signals would reflect off the ionosphere.

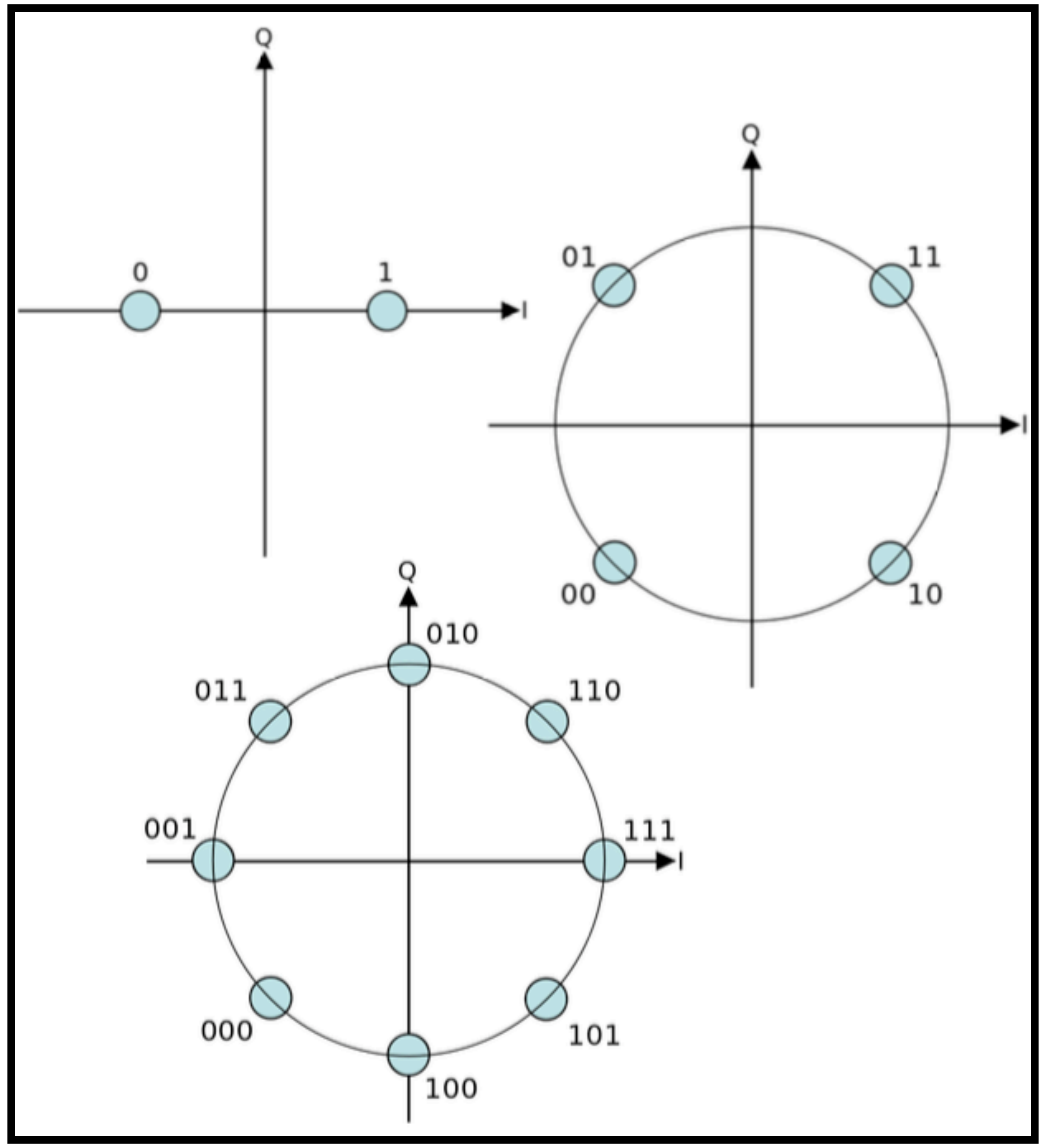
Digital modulations: ASK, FSK, PSK

In digital modulation, we use a finite number of analog signals (pulses) to represent pieces of the digital data.

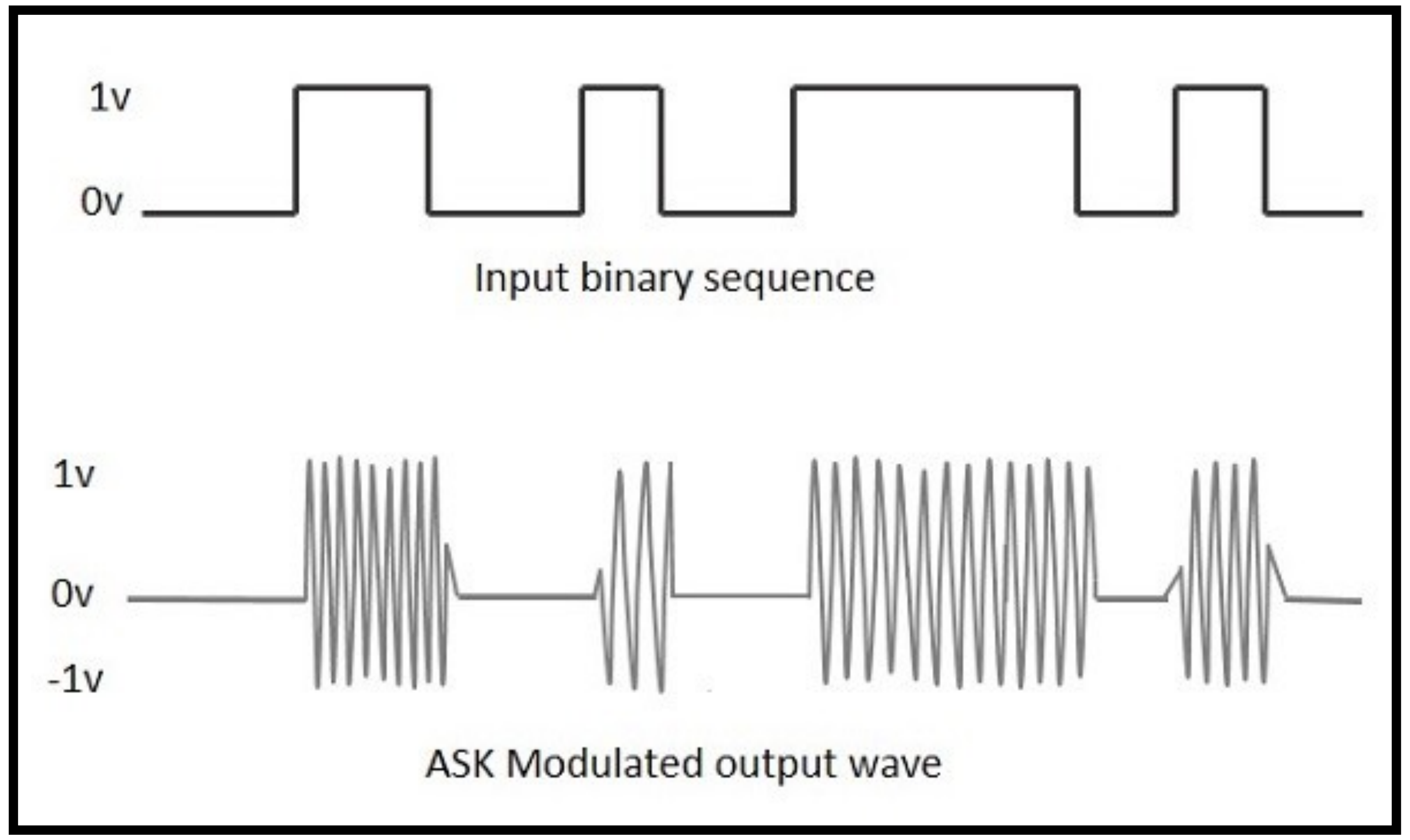
Frequency shift keying



Phase shift keying

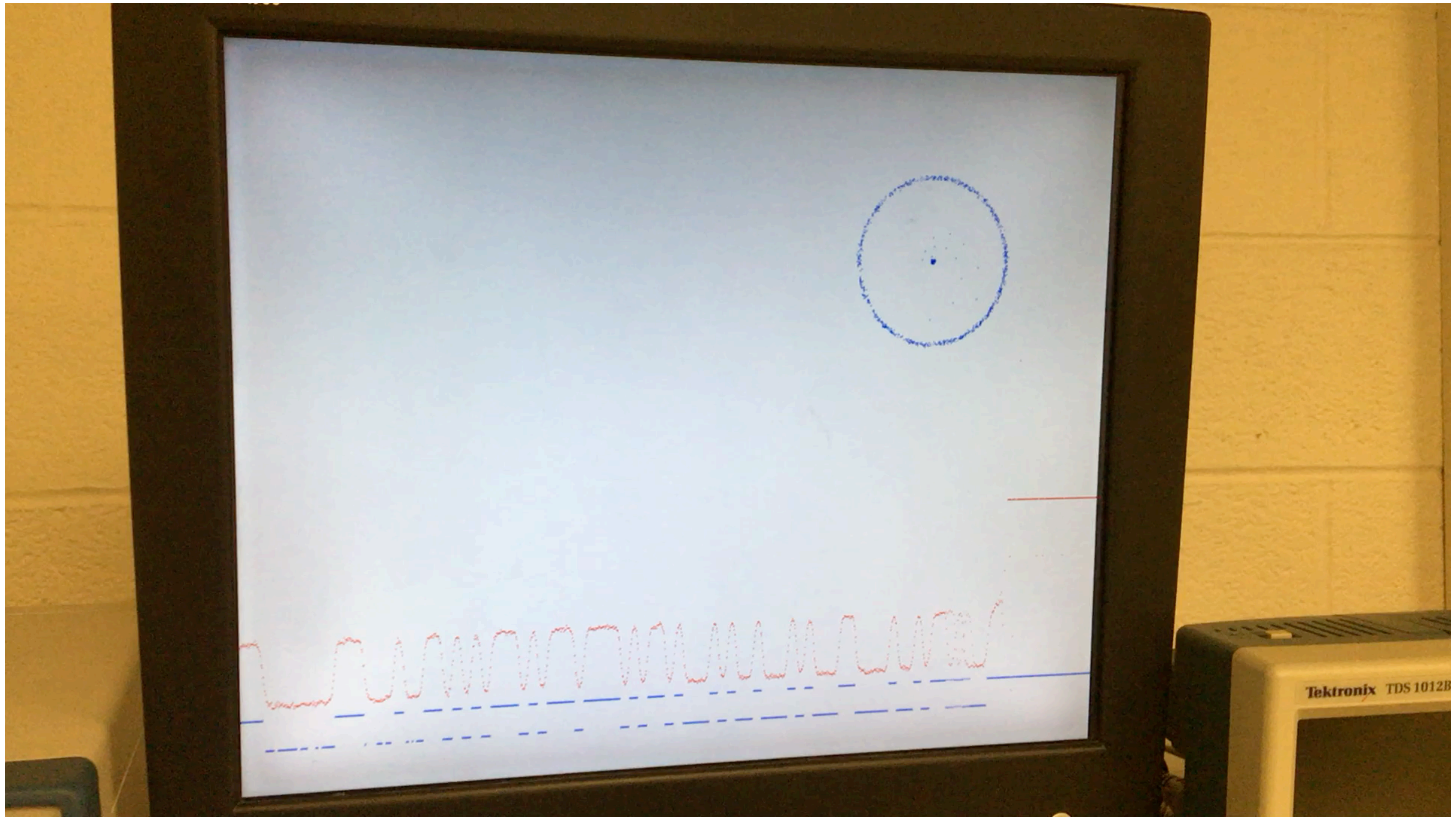


Amplitude shift keying



For each modulation, we could use more than two different analog signals to represent more than two pieces of digital information.

An aside: demodulating GFSK for Monarchs



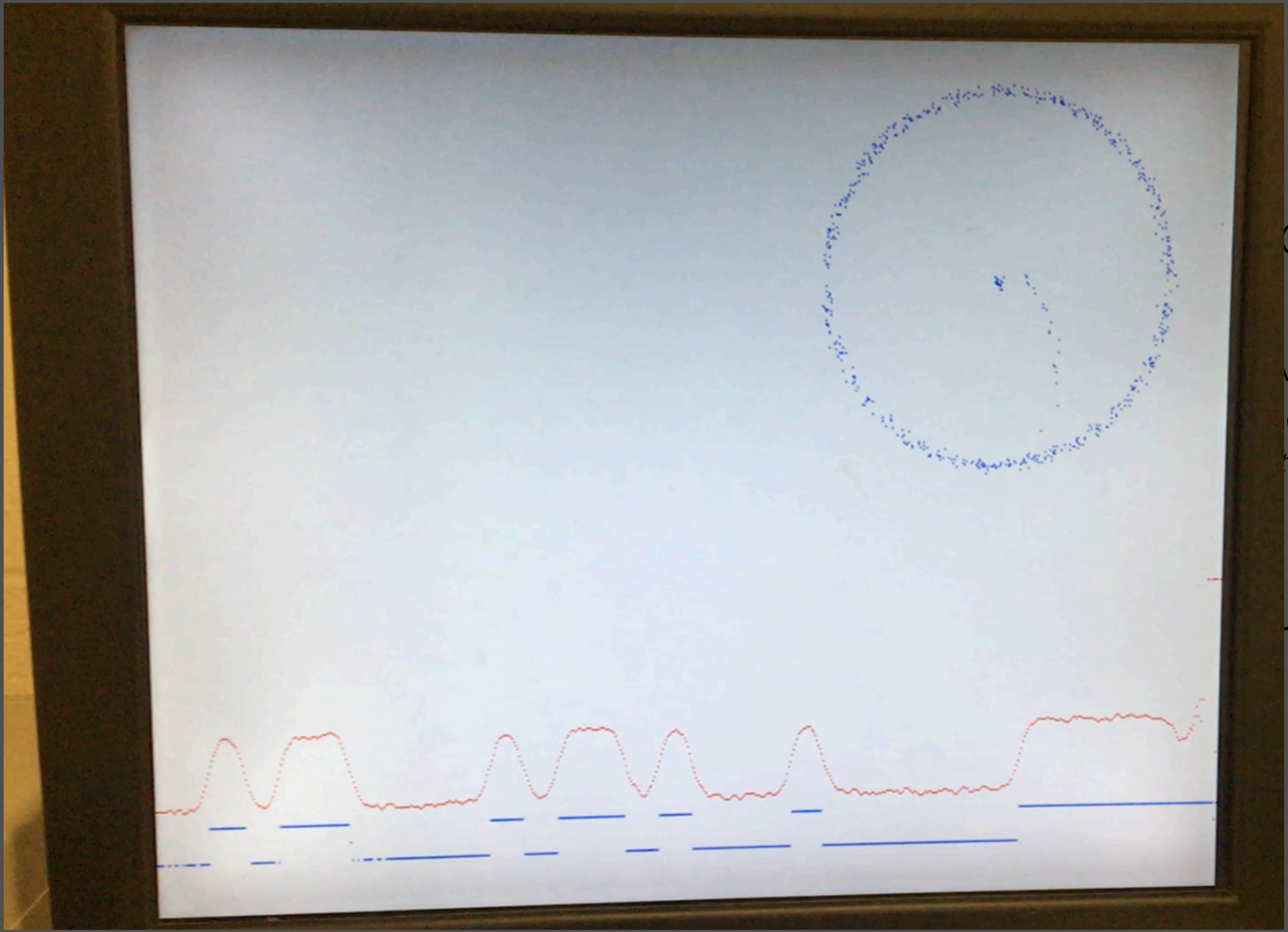
Realtime GFSK demodulation

1. Gather raw radio data (I/Q)

2. Approximate the derivative of the phase by finding the argument of the conjugate product of the n^{th} and $(n-1)^{st}$ samples:

$$y[n] = \textbf{arg}\,(x[n]\overline{x}[n-1])$$

3. Apply a binary slicer



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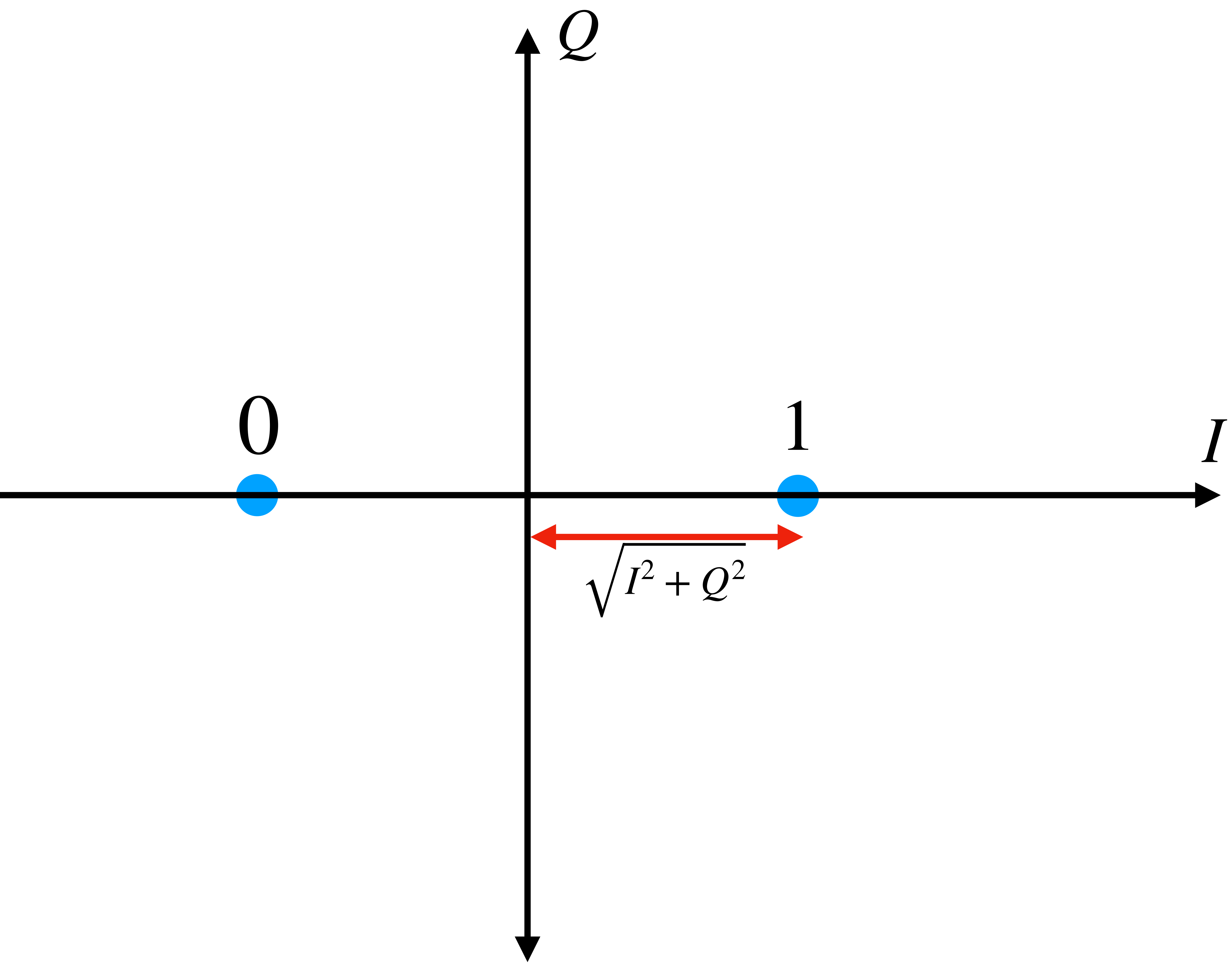
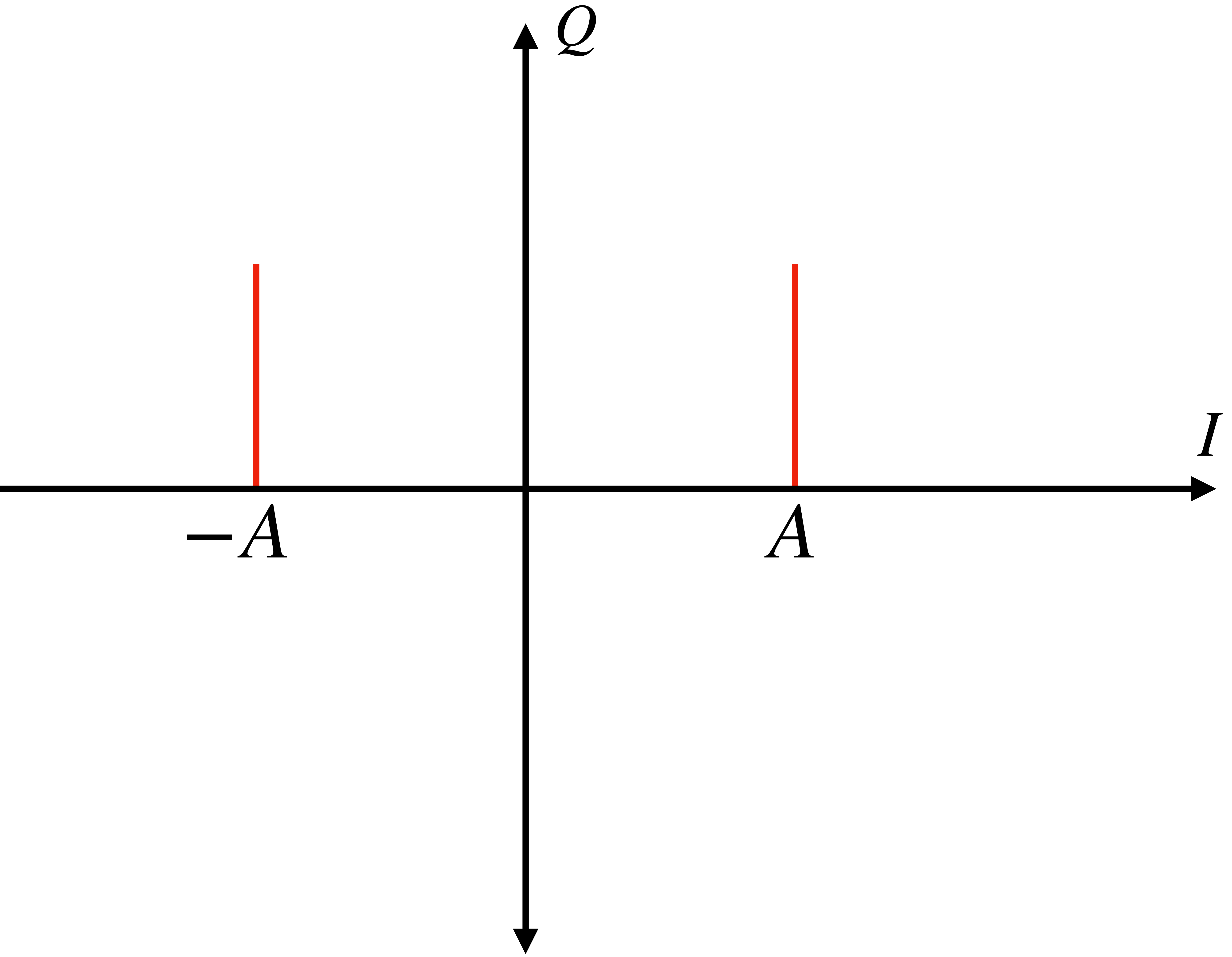
Binary phase shift keying



In BPSK, we encode 1's and 0's as two symbols that are 180 degrees out of phase, as shown left.

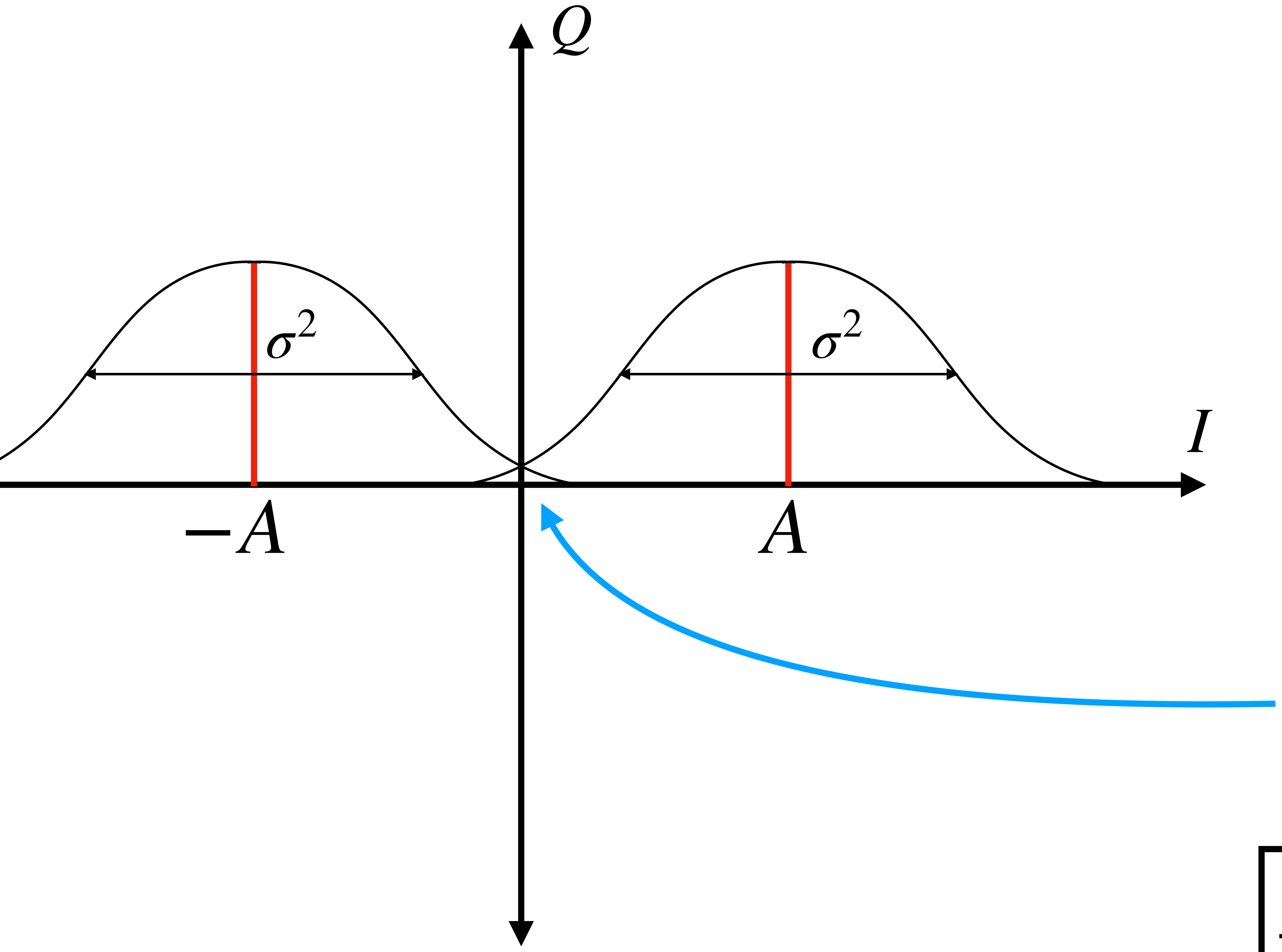
Binary phase shift keying: BER



In the absence of any noise, the symbols 1 and 0 would be a distance $\sqrt{I^2 + Q^2} = A$ from the origin.

Without any noise, a histogram of the received signal would look like that shown on the left.

Binary phase shift keying: BER

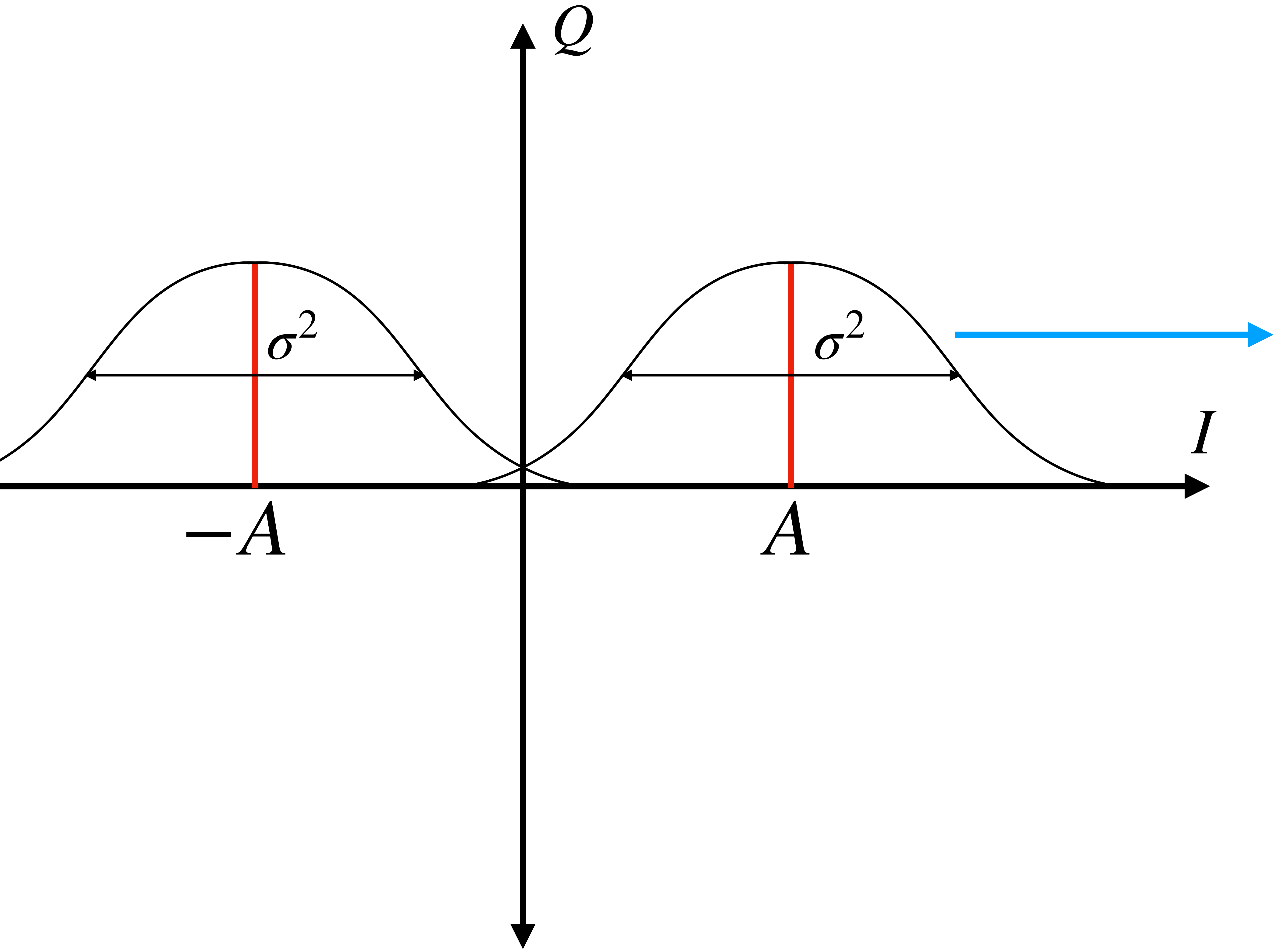


However, there is additive Gaussian noise on top of our signal. Instead of receiving perfectly distinct signals, we receive signal *plus* noise.

Note that these distributions overlap. It is possible to send the symbol for 1 and receive the signal for 0.

This can lead to *bit error.* In order to quantify the frequency of bit errors, we must understand our error distributions.

Binary phase shift keying: BER

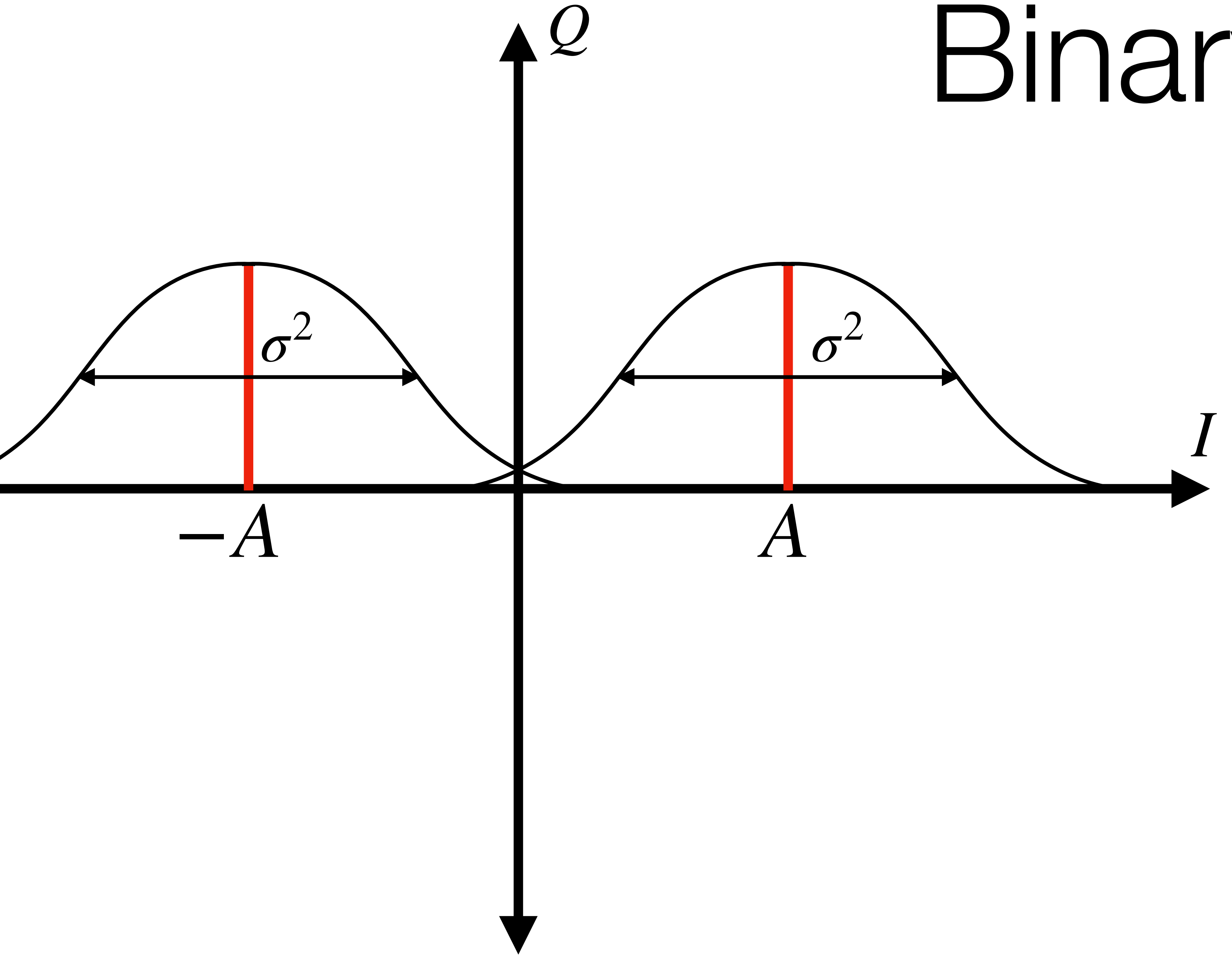


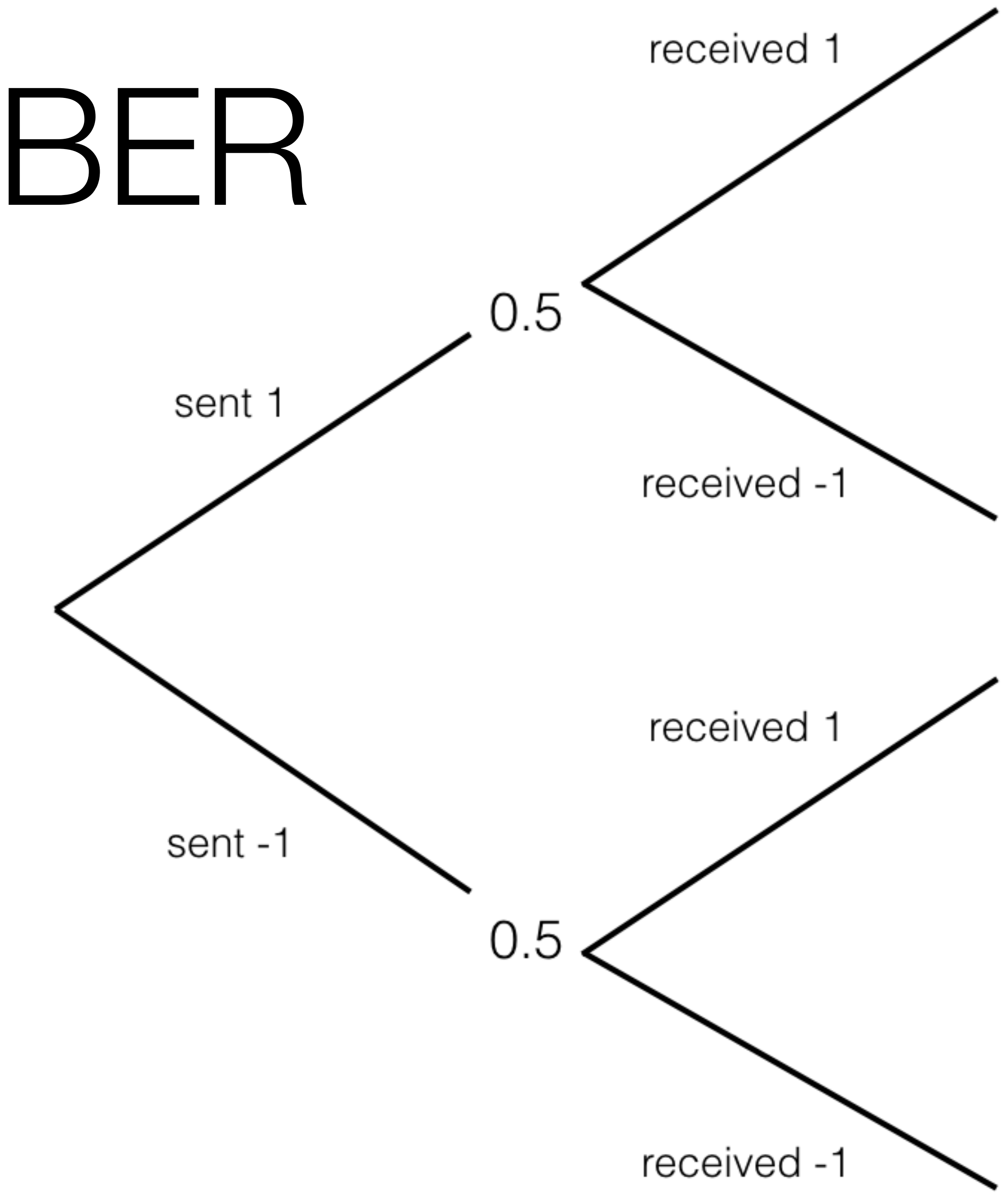
Zero-mean Gaussian noise. So, the variance is equal to the power of the noise.

$$\sigma^2 = \frac{P_R}{SNR} = \frac{N_0}{2}B$$

We can calculate with the analysis that we've already done. With the variance, we can calculate *bit error rate.*

Binary phase shift keying: BER





$$
\begin{aligned}
p(error) &= p\left(\textbf{transmit 0}\right) \cdot p\left(\textbf{receive 1} \mid \textbf{transmit 0}\right) + p\left(\textbf{transmit 1}\right) \cdot p\left(\textbf{receive 0} \mid \textbf{transmit 1}\right) \\
&= 0.5 \cdot p\left(\mathcal{N}(-A, \sigma^2) > 0\right) + 0.5 \cdot p\left(\mathcal{N}(A, \sigma^2) < 0\right) \\
&= p\left(\mathcal{N}(-A, \sigma^2) > 0\right) \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}\left(\frac{x+A}{\sigma}\right)^2} dx \longrightarrow \frac{1}{\sqrt{2\pi}} \int_{\frac{A}{\sigma}}^\infty e^{-\frac{1}{2}t^2} dt \equiv Q\left(\frac{A}{\sigma}\right)
\end{aligned}
$$

Binary phase shift keying: BER

$$p(error) = p\left(\textbf{transmit 0}\right) \cdot p\left(\textbf{receive 1} \mid \textbf{transmit 0}\right) + p\left(\textbf{transmit 1}\right) \cdot p\left(\textbf{receive 0} \mid \textbf{transmit 1}\right)$$

$$= 0.5 \cdot p\left(\mathcal{N}(-A, \sigma^2) > 0\right) + 0.5 \cdot p\left(\mathcal{N}(A, \sigma^2) < 0\right)$$

$$= p\left(\mathcal{N}(-A, \sigma^2) > 0\right)$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}\left(\frac{x+A}{\sigma}\right)^2} dx \longrightarrow \frac{1}{\sqrt{2\pi}} \int_{\frac{A}{\sigma}}^\infty e^{-\frac{1}{2}t^2} dt \equiv Q\left(\frac{A}{\sigma}\right)$$

Signal amplitude

Time associated w/ 1 bit

$$\frac{E_b}{N_0} = \frac{A^2 T_b}{N_0} = \frac{A^2 T_b}{2\sigma^2 T_b} = \frac{A^2}{2\sigma^2}$$

$$\boxed{p(error) = Q\left(\sqrt{2\frac{E_b}{N_0}}\right)} \longrightarrow$$

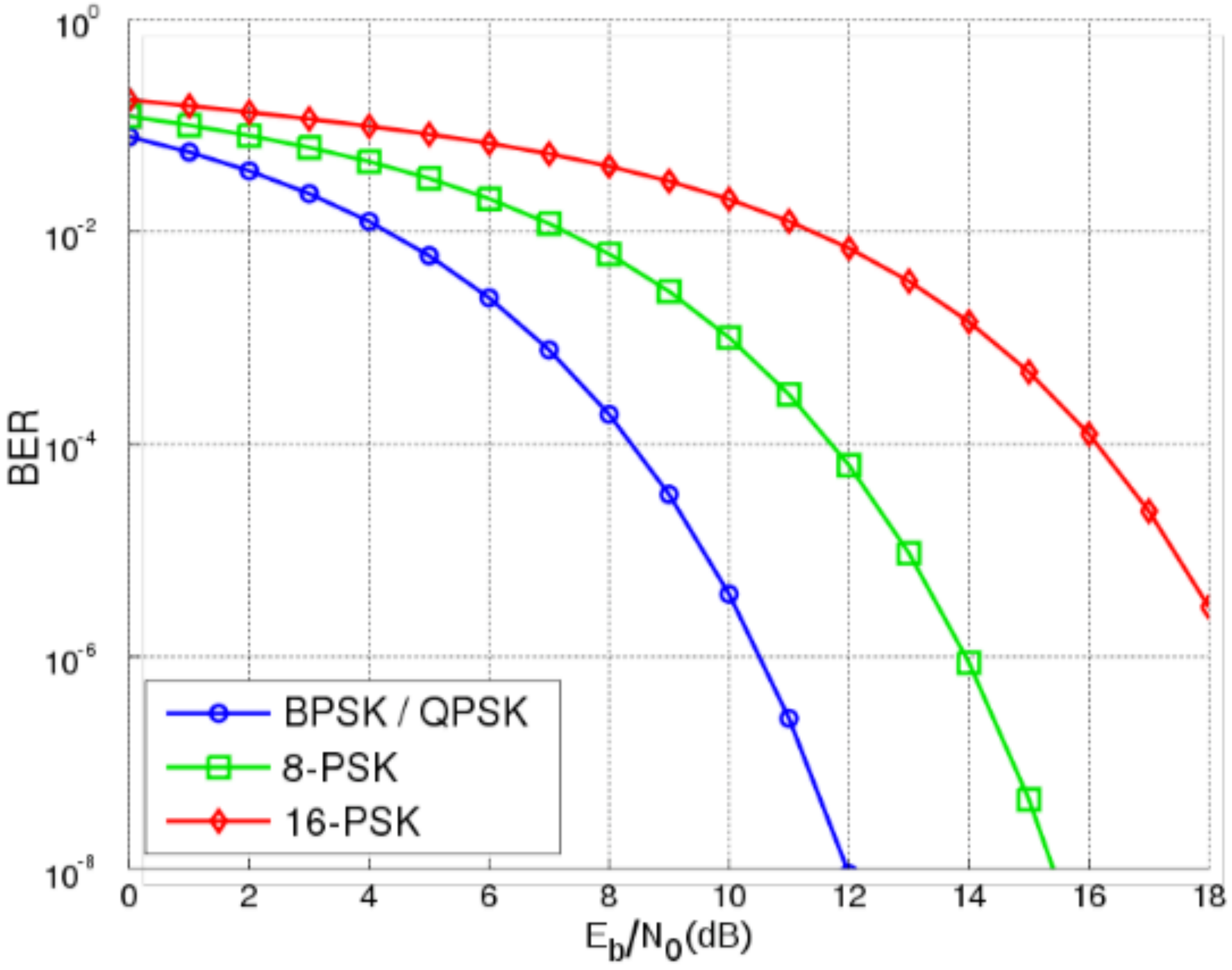
Given a required bit error rate, you can solve for the minimum allowable $\frac{E_b}{N_0}$

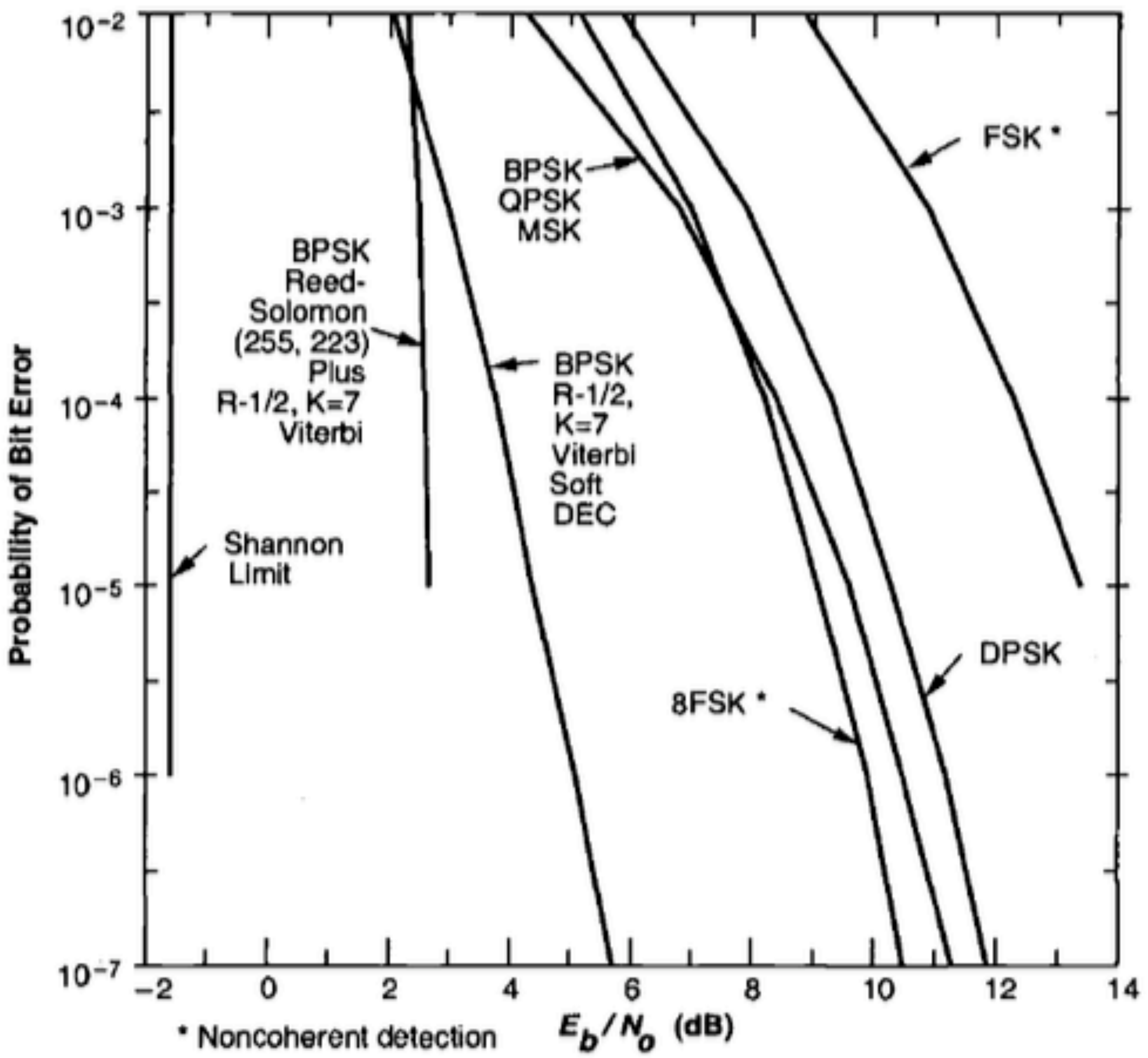
BER for other modulations

Via similar analysis:

$$BER_{QPSK} \approx Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$$

$$BER_{8PSK} \approx \frac{2}{3}Q\left(\sqrt{2\frac{E_b}{N_0}}\sin\frac{\pi}{8}\right)$$





Why not always use BPSK/QPSK? BER is not the only metric we care about. We also care about ***spectral efficiency***.

Spectral efficiency

Spectral efficiency is data rate per unit bandwidth, denoted by η.

bits/sec

bits/symbol (i.e. modulation choice)

$$\eta = \frac{R_b}{B} = \frac{log_2 M}{1 + \alpha}$$

Hz

rolloff factor

We will see that this places a second constraint on $\frac{E_b}{N_0}$.

Day 2

MAE 4160, 4161, 5160

V. Hunter Adams, PhD

Some comments about homework 1

- Which is better? *The spacecraft shall communicate at 27.6 kbps. The spacecraft shall communicate at 27.6 kbps +/- 100 bps.*
- Be careful of vague words like "safe."
- "Functioning" is a vague word.
- A spacecraft does not "permit" something. "Enable" is a good word.
- "Have mass less than" is better than "weigh." ("Weigh" on Earth, or on the destination planet/body?)

Brief reminder from last time . . .

Illustration of link budget equation parameters

$$\frac{E_b}{N_0} = \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2 K_B T_{sys} R_b}$$

The Link Budget Equation

We are interested in the ratio of signal energy to noise energy. We now have everything we need to compute that.

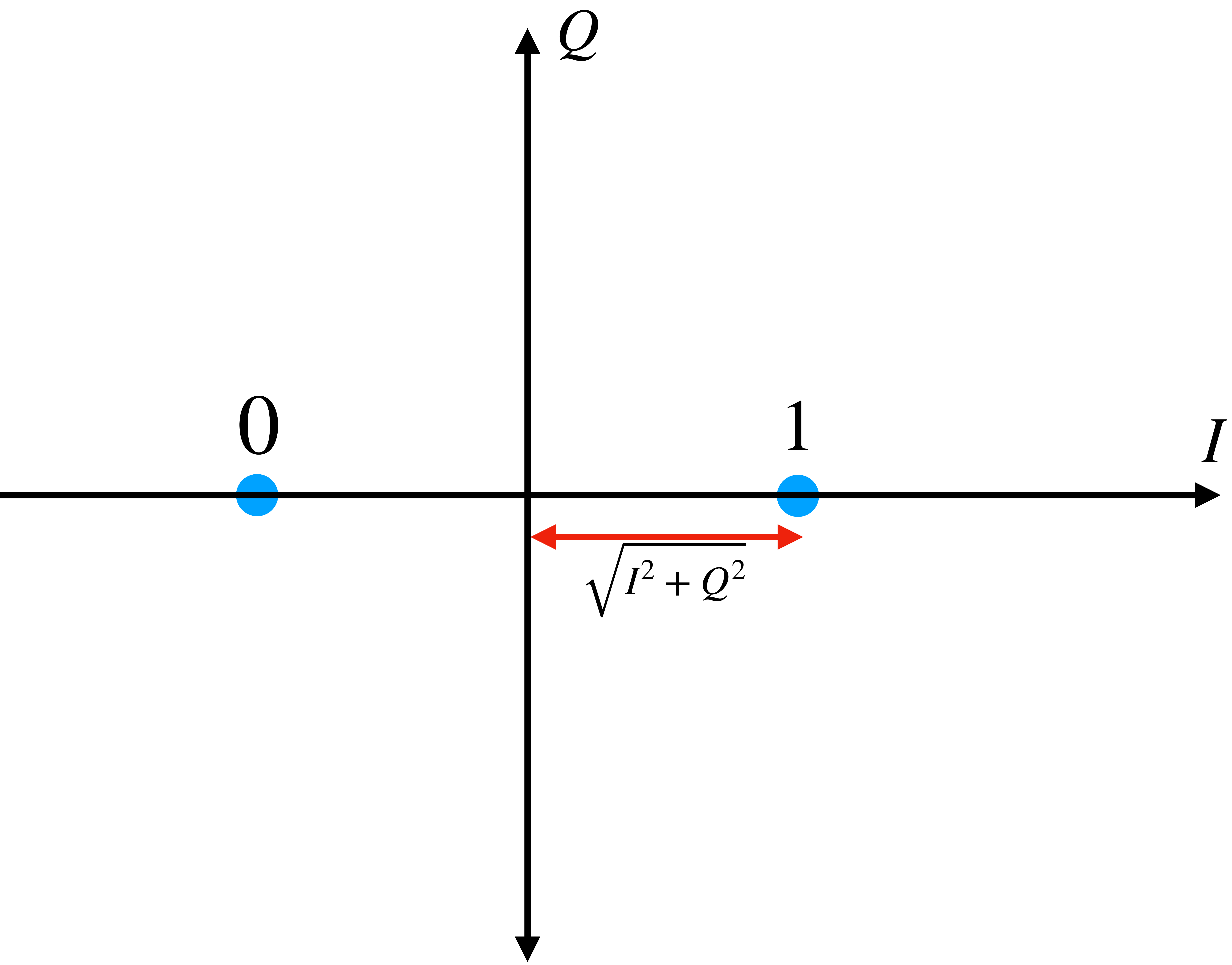
$$\frac{E_b}{N_0} = \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2 K_B T_{sys} R_b} \longrightarrow \textbf{signal to noise ratio per bit}$$

$$\left[\frac{E_b}{N_0}\right]_{db} = [P_T]_{db} + [G_T]_{db} + [G_R]_{dB} + [L_a]_{dB} + [L_l]_{dB} + 10\log_{10}\left[\left(\frac{\lambda}{4\pi R}\right)^2\right] - 10\log_{10}\left(K_B T_{sys} R_b\right)$$

$$\frac{P_R}{P_N} = \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2 K_B T_{sys} B} \longrightarrow \textbf{signal to noise ratio per bandwidth}$$

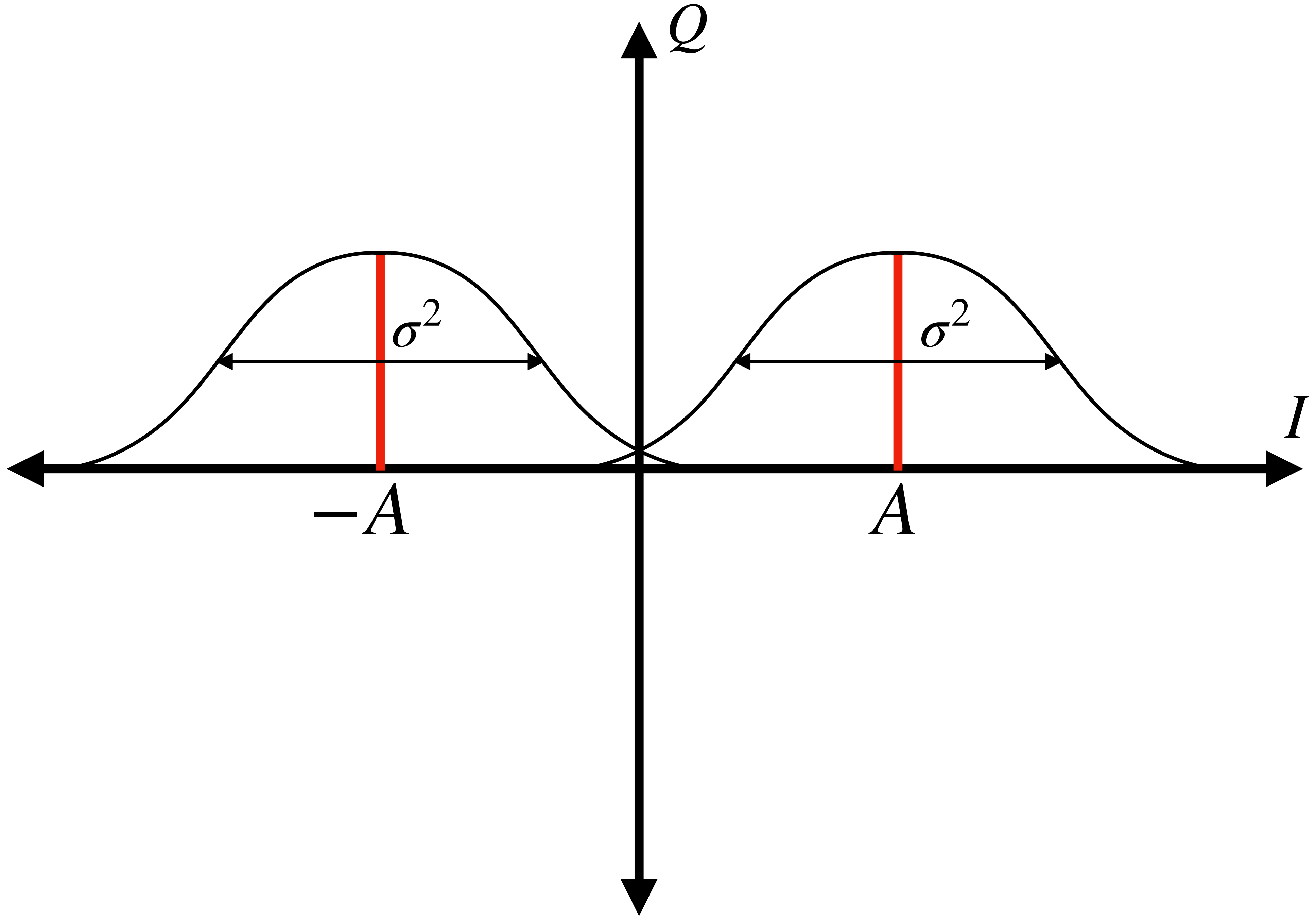
$$\left[\frac{S}{N}\right]_{db} = [P_T]_{db} + [G_T]_{db} + [G_R]_{dB} + [L_a]_{dB} + [L_l]_{dB} + 10\log_{10}\left[\left(\frac{\lambda}{4\pi R}\right)^2\right] - 10\log_{10}\left(K_B T_{sys} B\right)$$

Bit error rate for BPSK



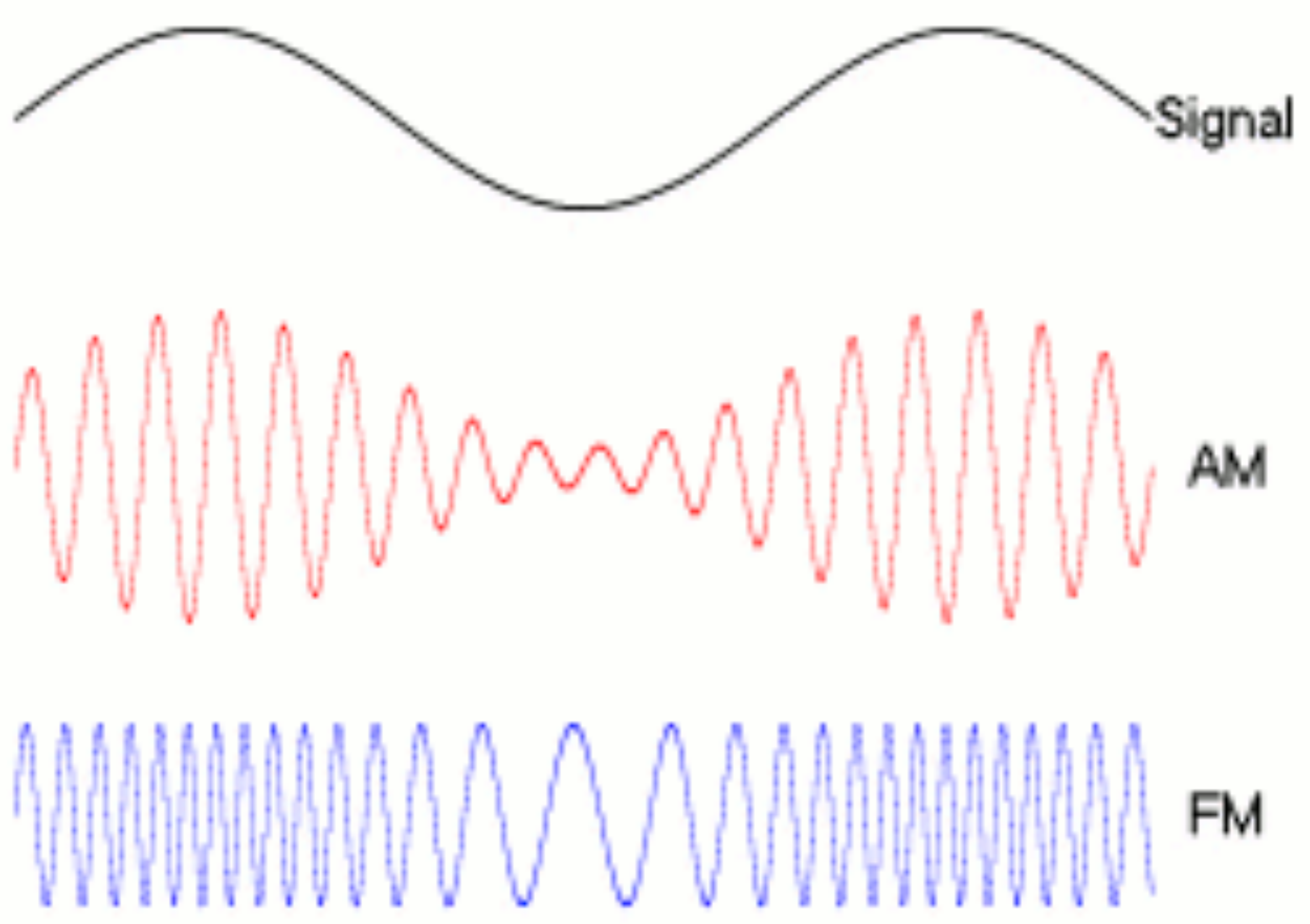
In BPSK, we encode 1's and 0's as two symbols that are 180 degrees out of phase, as shown left.

Bit error rate for BPSK



$$
\begin{aligned}
p(error) &= p\left(\textbf{transmit 0}\right) \cdot p\left(\textbf{receive 1} \mid \textbf{transmit 0}\right) + p\left(\textbf{transmit 1}\right) \cdot p\left(\textbf{receive 0} \mid \textbf{transmit 1}\right) \\
&= 0.5 \cdot p\left(\mathcal{N}(-A, \sigma^2) > 0\right) + 0.5 \cdot p\left(\mathcal{N}(A, \sigma^2) < 0\right) \\
&= p\left(\mathcal{N}(-A, \sigma^2) > 0\right) \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int_0^{\infty} e^{-\frac{1}{2}\left(\frac{x+A}{\sigma}\right)^2} dx \longrightarrow \frac{1}{\sqrt{2\pi}} \int_{\frac{A}{\sigma}}^{\infty} e^{-\frac{1}{2}t^2} dt \equiv Q\left(\frac{A}{\sigma}\right) = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) \longrightarrow \boxed{\textbf{Establishes } \frac{E_b}{N_{0_{min}}}}
\end{aligned}
$$

Analog modulations: AM and FM

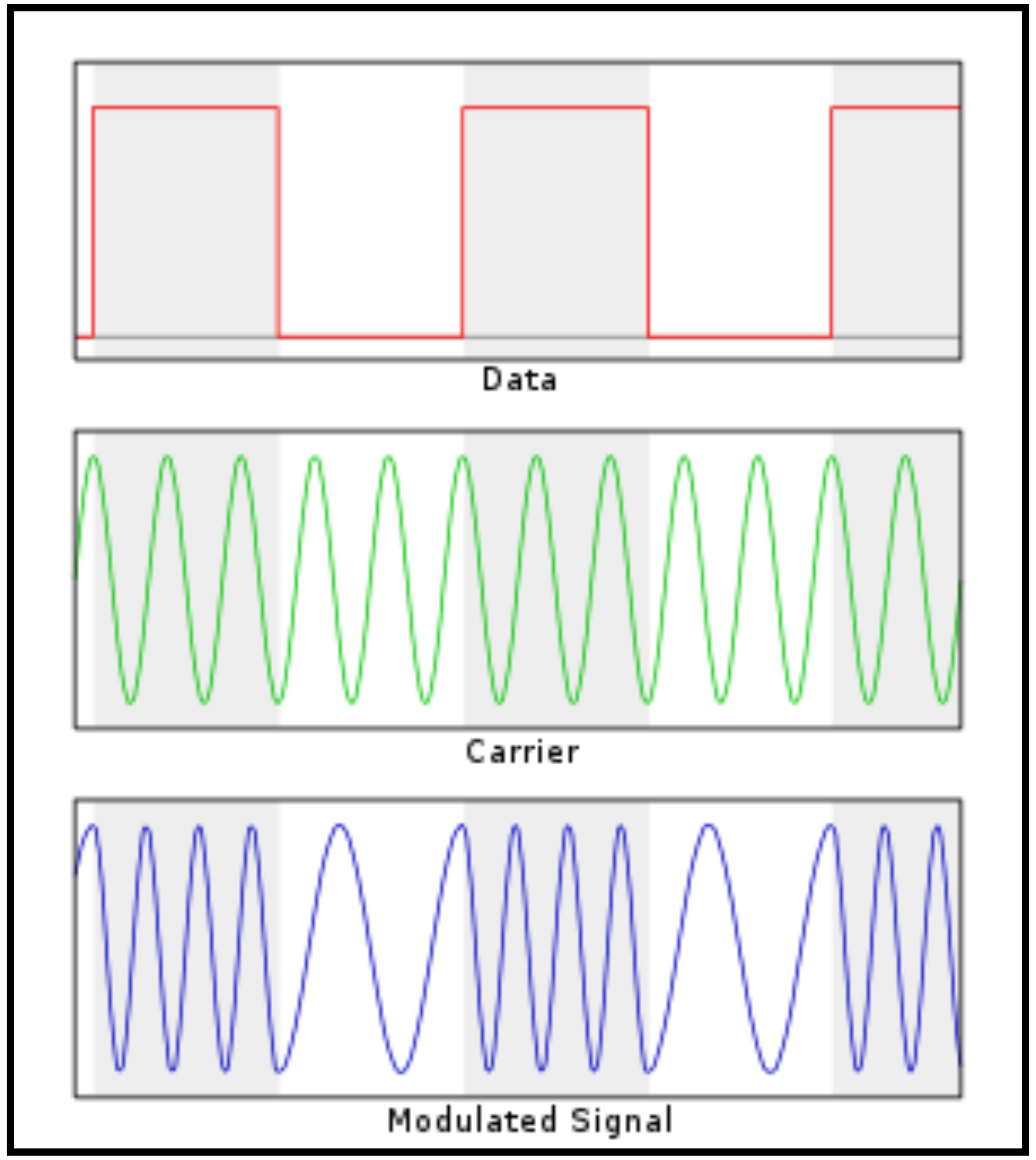


We need modulation because we must carry our low-frequency data on high-frequency waves. Otherwise, we'd require huge antennas, and our signals would reflect off the ionosphere.

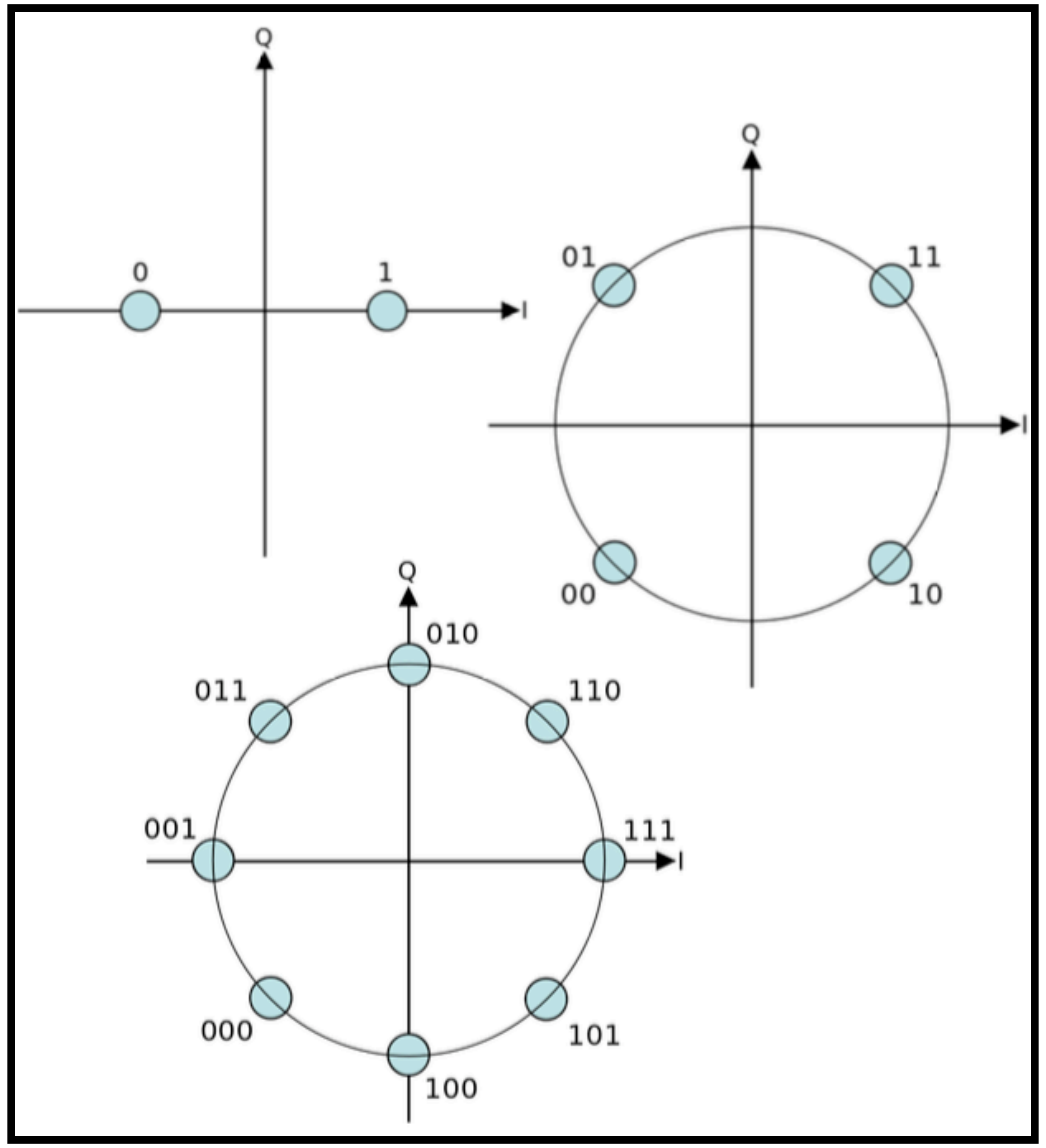
Digital modulations: ASK, FSK, PSK

In digital modulation, we use a finite number of analog signals (pulses) to represent pieces of the digital data.

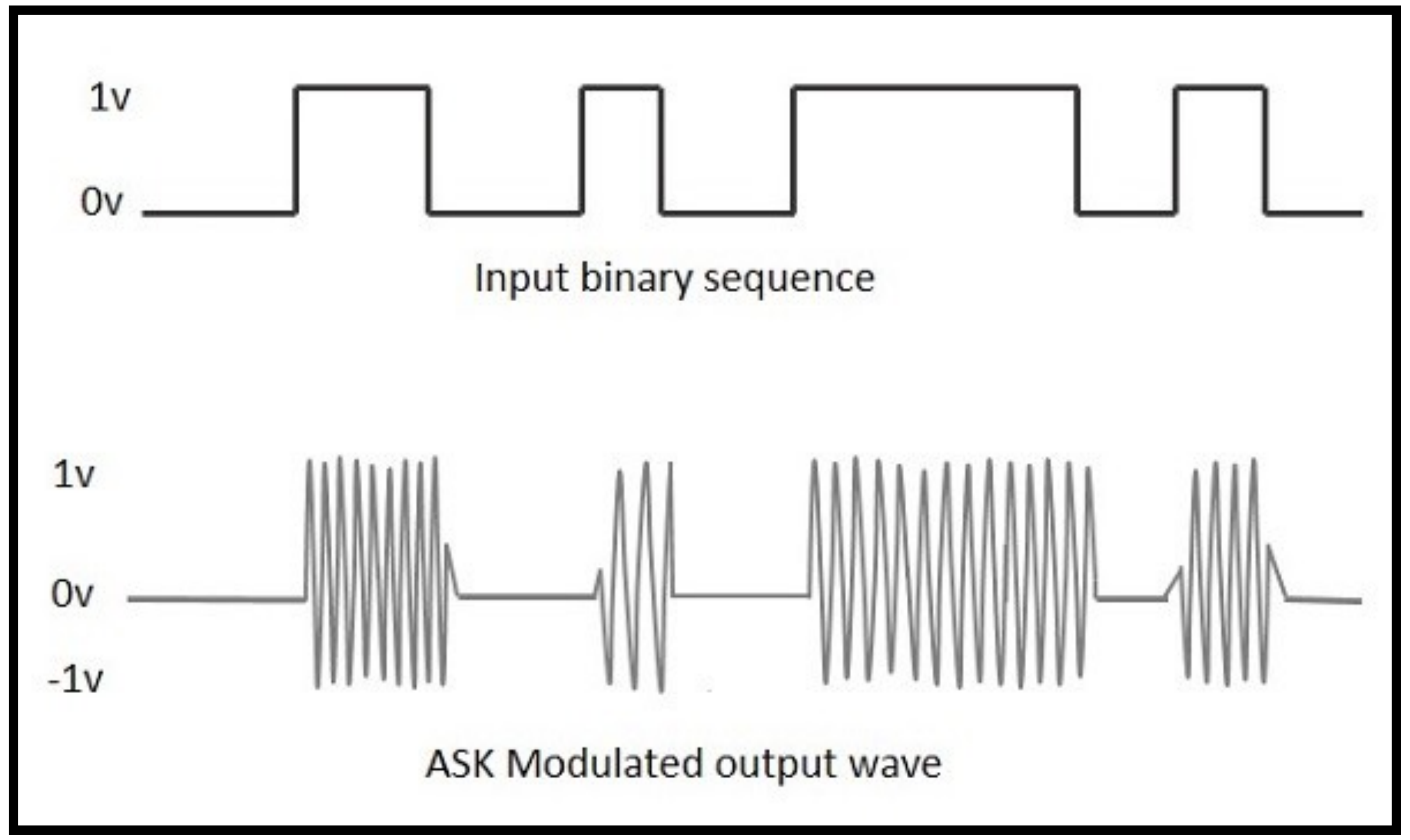
Frequency shift keying



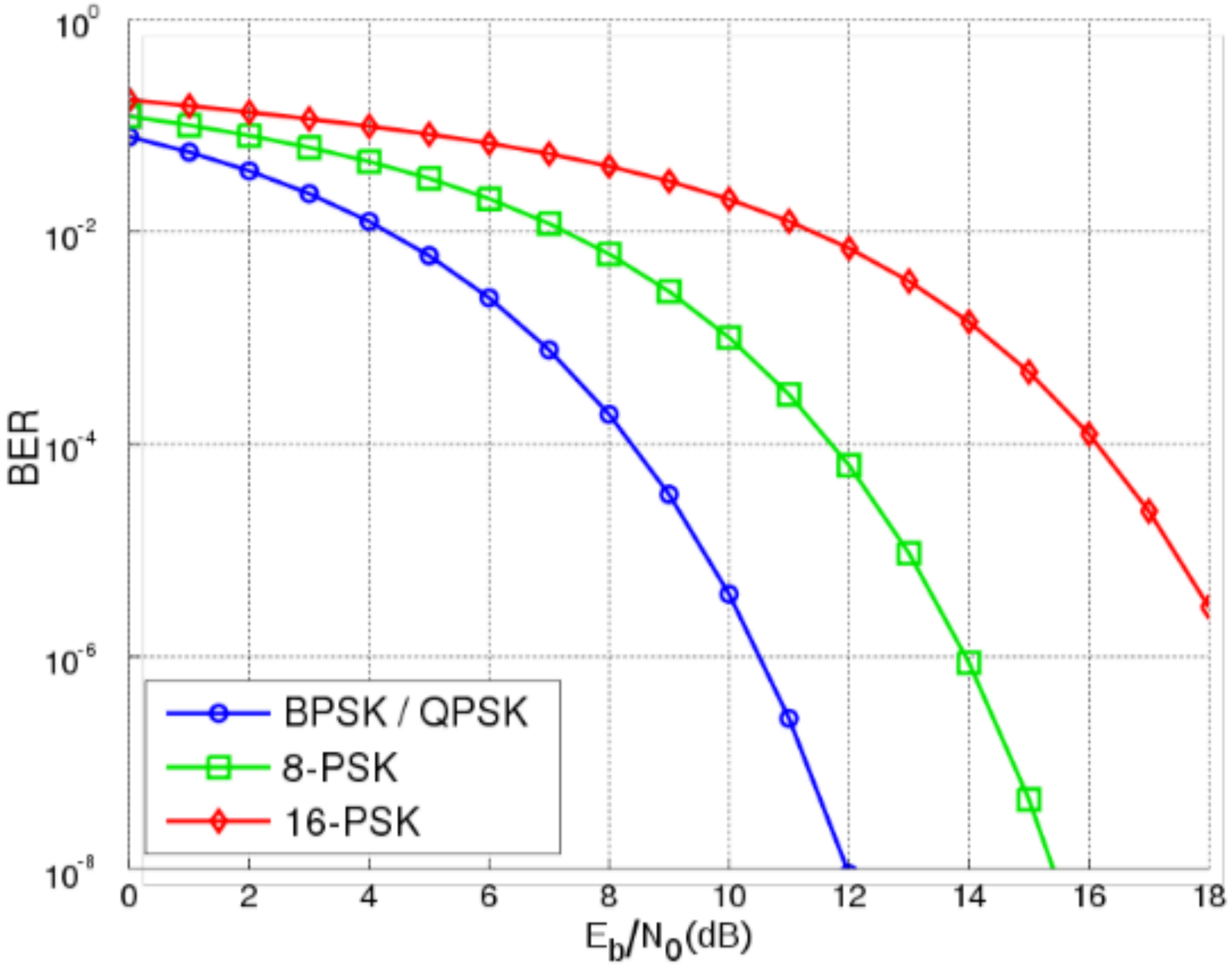
Phase shift keying

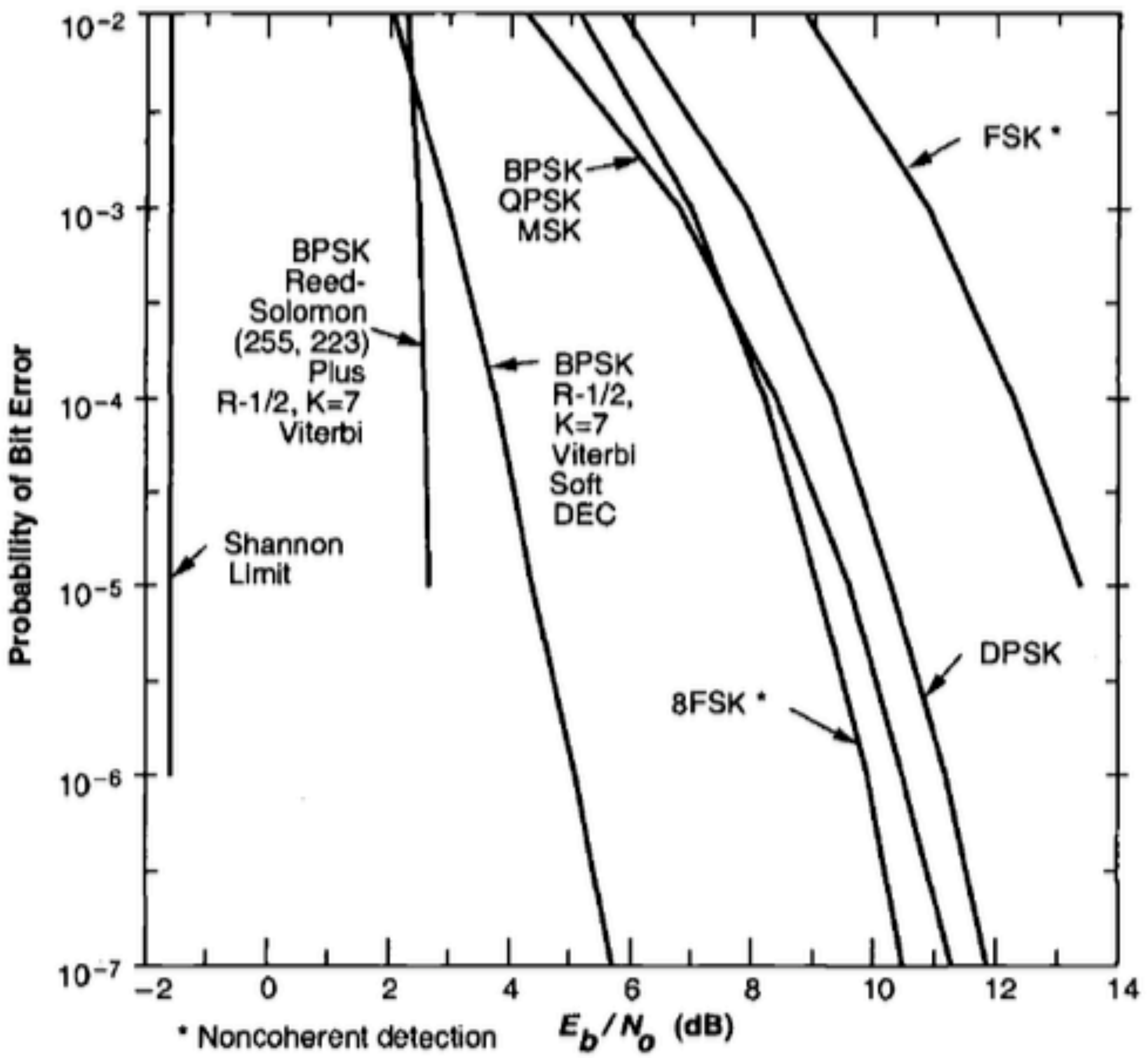


Amplitude shift keying



For each modulation, we could use more than two different analog signals to represent more than two pieces of digital information.





Why not always use BPSK/QPSK? BER is not the only metric we care about. We also care about ***spectral efficiency***.

Spectral efficiency

Spectral efficiency is data rate per unit bandwidth, denoted by η.

bits/sec

bits/symbol (i.e. modulation choice)

$$\eta = \frac{R_b}{B} = \frac{log_2 M}{1 + \alpha}$$

Hz

rolloff factor

We will see that this places a second constraint on $\frac{E_b}{N_0}$.

What spectral efficiency do I require to send 100 kbps through a 20 kHz channel?

What is the greatest data rate we could **possibly** achieve?

Today’s topics:

- Link budget equation
- Modulations
- Bit error rate
- **Shannon Limit**
- Coding techniques
- Antennas
- Examples

The Shannon Limit

Establishes the channel capacity for the communication link, which is a bound on the maximum amount of error-free information that can be transmitted per unit time with a specified bandwidth. It is assumed that the signal power is bounded, and that the Gaussian noise is characterized by a known power or power spectral density.

$$\boxed{R_b \leq C = B \log_2\left(1 + \frac{S}{N}\right)}$$

or, alternatively

$$\boxed{\frac{E_b}{N_0} \geq \frac{2^\eta - 1}{\eta}}$$

$R_b =$ **data rate (bps)**

$C =$ **channel capacity (bits/second)**

$B =$ **bandwidth of the channel (Hz)**

$S =$ **average received signal power over the bandwidth (watts)**

$N =$ **average power of the noise and interference over the bandwidth (watts)**

$\frac{S}{N} =$ **signal to noise ratio**

What is the minimum $\dfrac{E_b}{N_0}$ that I require to send 100 kpbs through a 20 kHz channel?

$$\eta = \frac{R_b}{B} = \frac{100{,}000}{20{,}000} = 5$$

$$\frac{E_b}{N_0} \geq \frac{2^\eta - 1}{\eta} = \frac{2^5 - 1}{5} = 6.2$$

1. Establish requirements for data rate R_b and bit error rate BER based on mission objectives.

2. Choose (or get assigned by a regulatory agency) a frequency $f = \frac{c}{\lambda}$ and a bandwidth B.

3. Given your bandwidth B and your required data rate R_b, solve for your required spectral efficiency $\eta = \frac{R_b}{B}$.

4. Use the Shannon Limit equation to solve for the minimum required $\frac{E_b}{N_0}$ to achieve that spectral efficiency.

$$\frac{E_b}{N_0}_{min} \geq \frac{2^\eta - 1}{\eta}$$

5. For each candidate modulation, calculate the minimum required $\frac{E_b}{N_0}$ to achieve the required bit error rate.

$$p(error) = Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$$

(e.g., for BPSK)

6. If $\frac{E_b}{N_0}_{min}$ for the required BER is greater than that from the Shannon limit, *either* use **coding** to drop the BER for a particular $\frac{E_b}{N_0}$, *or* increase your $\frac{E_b}{N_0}_{min}$.

8. Use the link budget equation to design your system such that the system's $\frac{E_b}{N_0} > \frac{E_b}{N_0}_{min}$, plus some margin.

$$\frac{E_b}{N_0} = \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2 K_B T_{sys} R_b}$$

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Big design space!

$$\frac{E_b}{N_0} = \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2 K_B T_{sys} R_b}$$

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Big design space!

$$\frac{E_b}{N_0} = \frac{P_T G_T G_R L_a L_l \lambda^2}{(4\pi d)^2 K_B T_{sys} R_b}$$

Note that there is a lot of coupling between your design parameters. Typically, a link budget is organized in a spreadsheet so that we can change a particular parameters and observe the effect on all other parameters.

Monarchs: an extreme example

Suppose a Monarch in a 400 km orbit. What is the maximum theoretical data rate from this spacecraft to a handheld ground station?

$$\lambda = \frac{c}{f} = \frac{299{,}792{,}458}{915{,}000{,}000} = 0.32m$$

$G_T = 0dbW$ **(isotropic)**

$$d = 400{,}000m$$

$$P_t = 10mW = -20dbW$$

$$G_R = 7dbW$$

$$T_{sys} \approx 150K$$

$$B = 100kHz$$

Atmospheric/line losses $= -5dbW$

$$\left[\frac{S}{N}\right]_{db} = [P_T]_{db} + [G_T]_{db} + [G_R]_{dB} + [L_a]_{dB} + [L_l]_{dB} + 10\log_{10}\left[\left(\frac{\lambda}{4\pi R}\right)^2\right] - 10\log_{10}\left(K_B T_{sys} B\right)$$
$$= -5.08$$

convert from dBW back to W

$$\frac{S}{N} = 10^{\frac{\left[\frac{S}{N}\right]_{dB}}{10}} = 0.31$$

Shannon limit

$$R_b \leq C = B\log_2\left(1 + \frac{S}{N}\right)$$
$$= 100{,}000 \cdot \log_2(1 + 0.31)$$
$$= 38.956kbps$$

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How do we approach this limit?

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- Shannon Limit
- **Coding techniques**
- Antennas
- Examples

Source vs. channel coding

Source coding and **channel coding** are two different kinds of codes used in digital communications systems.

They have orthogonal goals:

- **Source Coding:** data compression (decrease data rate)
- **Channel Coding:** error detection and correction (by increasing the data rate)

Source coding vocabulary

Lossy: Some information is lost, and perfect reconstruction of the original data is not possible, but a much higher data reduction is achieved. Useful when bit rate reduction is very important and integrity is not critical (jpg, mp3).

Lossless: Allows perfect reconstruction of the original signal. Lossless techniques are used when it is essential to maintain the integrity of the data, and many scientists will push for this (zip, png).

Typically exploit the **structure of the data**. Are there long stretches of 1's or 0's? Are there certain combinations of bits that are more likely than others?

Run-length encoding: a lossless source code

$$[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1] \longrightarrow [1,14,2]$$

useful for black and white images, for example

Huffman coding: another lossless source code

If there are some symbols which are more likely than others, we can use fewer bits to encode the more likely combinations. Assign 0 to the most likely symbol, the others start with 1. Then assign 10 to the next most likely symbol, the others start with 11. Etc.

With these prefixes, there is no ambiguity about where one symbol ends and the next one starts. Suppose the following symbols:

$a1$: 0
$a2$: 10
$a3$: 110
$a4$: 111

$$0010111100101010110100 \longrightarrow a1, a1, a2, a4, a2, a1, a2, a2, a3, a2, a1$$

Channel coding

Channel codes at **redundancy bits** in order to detect and correct errors.

Detecting errors: Suppose that we added a *parity bit* at the end of every N bits so that the sum of all the bits including the parity bit is always 0. Then we can detect one error:

0101010**1** $\rightarrow$ **OK, no errors. (or there could be 2 errors . . .)**

1110110**0** $\rightarrow$ **NOK. There's an error (or 3 . . .), but where is it?**

Channel coding

Channel codes at **redundancy bits** in order to detect and correct errors.

Correcting errors: Imagine that we simply transmit each bit 3 times. Then there are two possible symbols: 000 and 111. We say that the code has a *distance* of 3, because 3 bits need to change in order to change a valid symbol into another. This code can detect 2 errors and correct 1, assuming that 2 bit flips are much less likely than one bit flip:

Receive 100, 010, 001 $\longrightarrow$ Correct to 000 (much more likely than 111)

Receive 110, 101, 011 $\longrightarrow$ Correct to 111 (much more likely than 000)

Channel coding

Forward error correcting codes embed the necessary information in the transmission to detect and correct errors. A particular FEC is specified by the following properties:

Distance: Minimum number of bits needed to transform between two valid symbols

Rate: Number of data bits/total number of bits

Code gain: Gain in dB in link budget equation for equal BER.

Hamming Codes

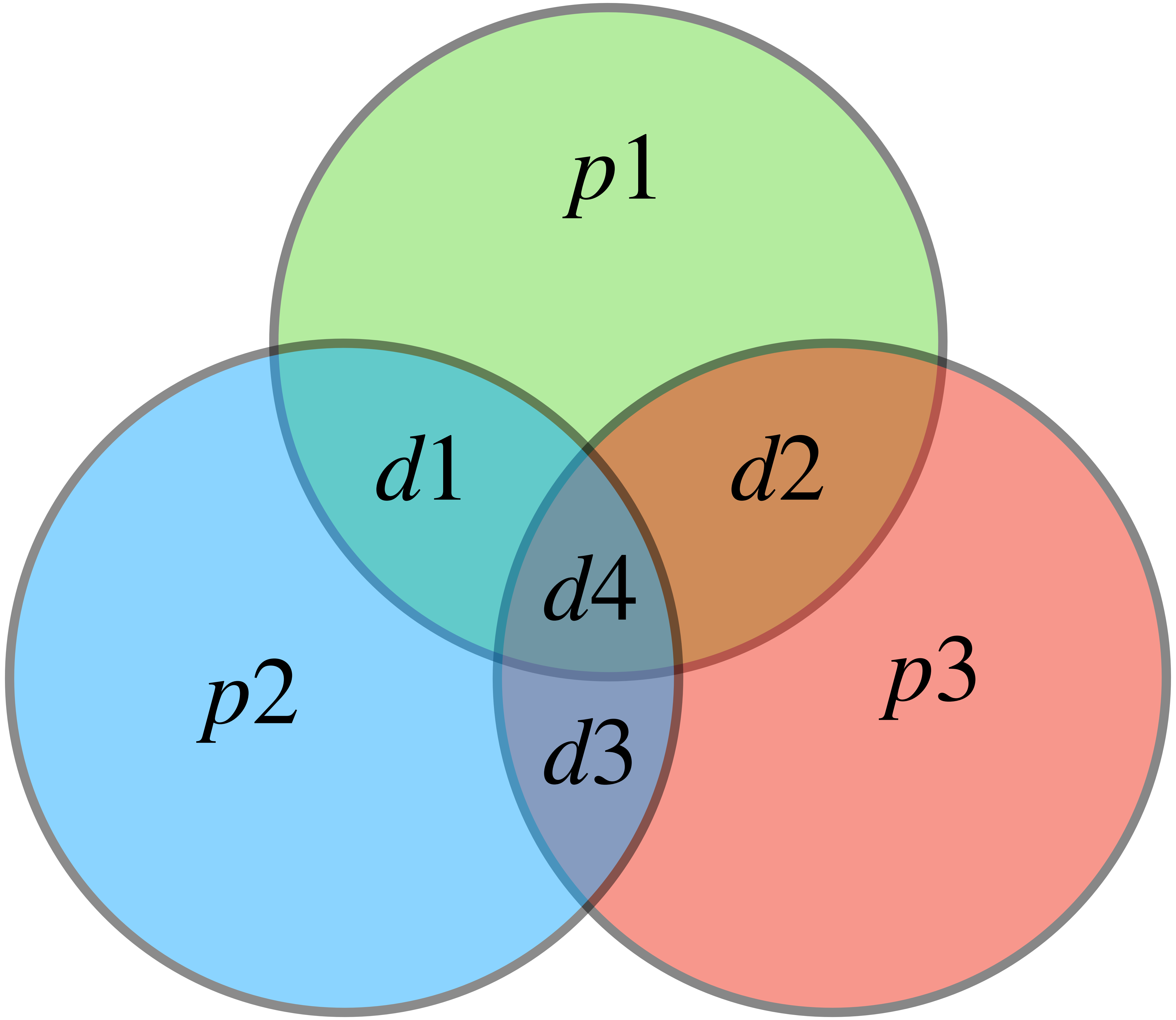
- Hamming codes are distance 3. They can detect 2 errors and correct 1.
- Hamming notation: $(2^r - 1, 2^r - r - 1)$. E.g. Hamming (7,4) adds 3 bits of redundancy for every 4 bits, for a rate of 4/7
- Parity bits are added at positions 1,2,4,8,…. The rest are data bits.

For a (7,4) Hamming code:

$$[d1 \quad d2 \quad d3 \quad d4] \longrightarrow \left[\mathbf{p1} \quad \mathbf{p2} \quad d1 \quad \mathbf{p3} \quad d2 \quad d3 \quad d4\right]$$

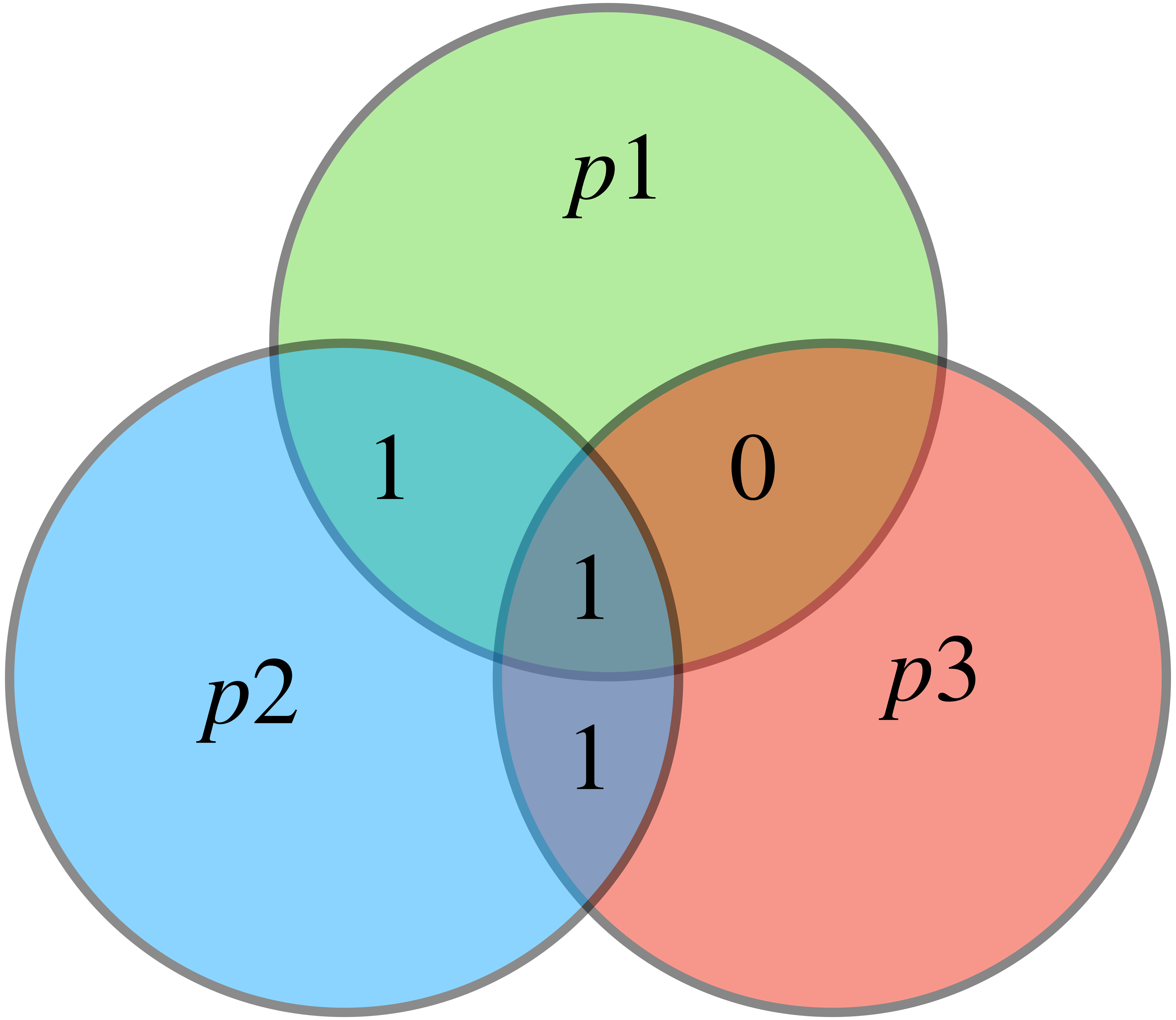
These parity bits are calculated according to the following rule:

$$[d1 \quad d2 \quad d3 \quad d4] \longrightarrow [\mathbf{p1} \quad \mathbf{p2} \quad d1 \quad \mathbf{p3} \quad d2 \quad d3 \quad d4]$$



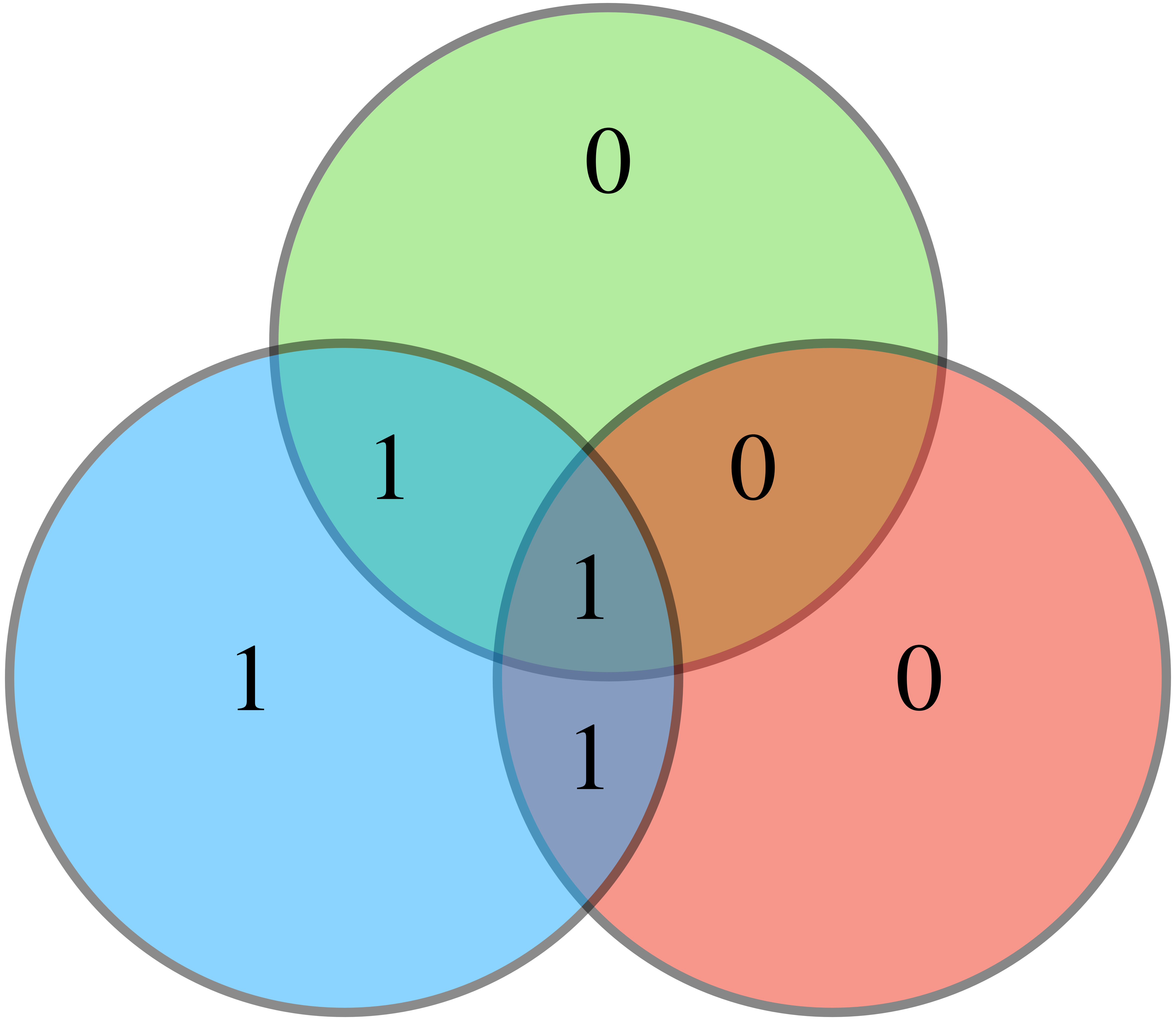
1. Start by filling in each data bit d
2. Assign each parity bit such that the sum of all bits in each circle is even

$$[1 \quad 0 \quad 1 \quad 1] \longrightarrow [\mathbf{p1} \quad \mathbf{p2} \quad 1 \quad \mathbf{p3} \quad 0 \quad 1 \quad 1]$$



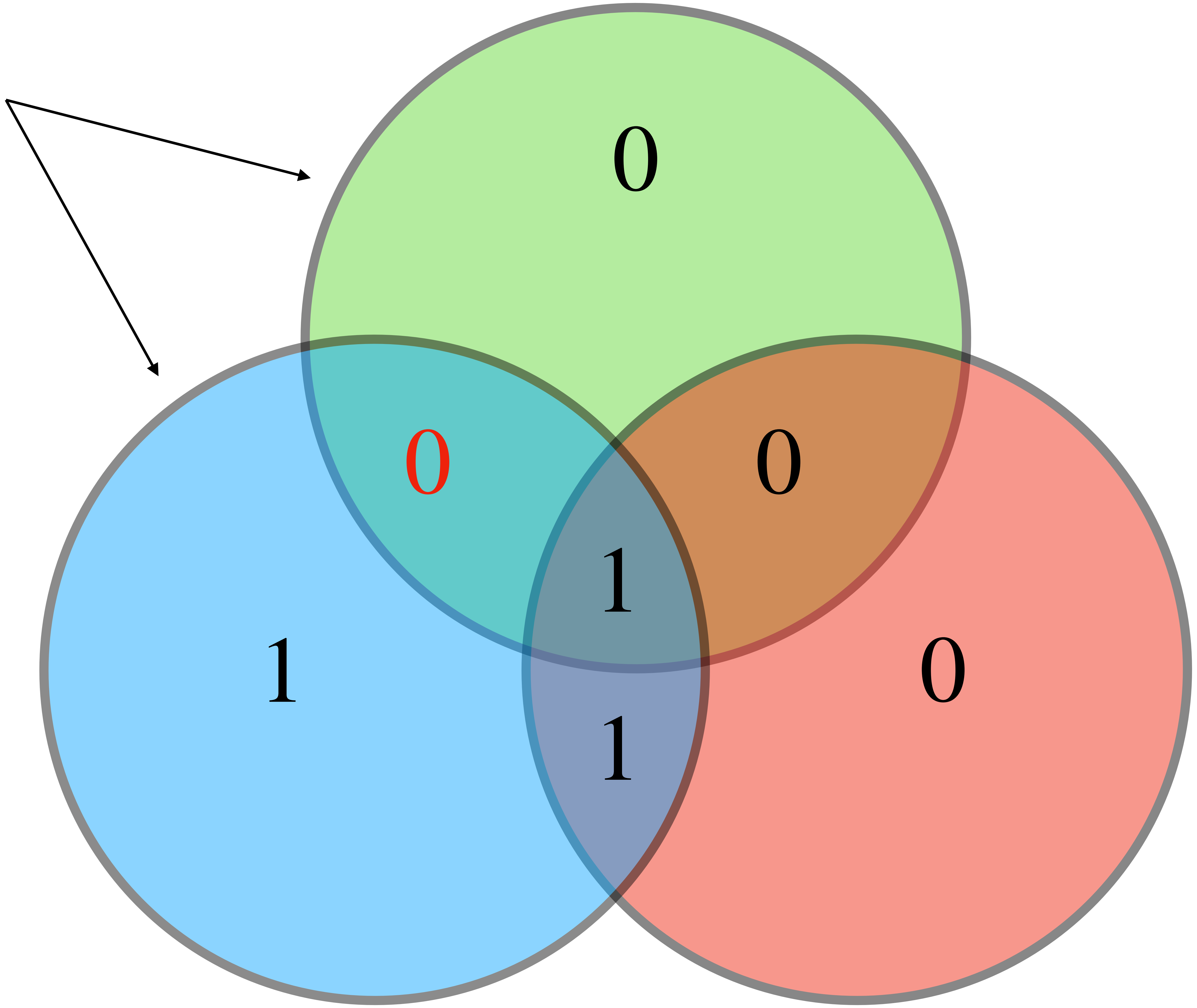
What should each parity bit be?

$$[1 \quad 0 \quad 1 \quad 1] \longrightarrow [0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1]$$



If any particular bit got flipped, which circles would have their sums affected?

$$[1 \quad 0 \quad 1 \quad 1] \longrightarrow [0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1]$$



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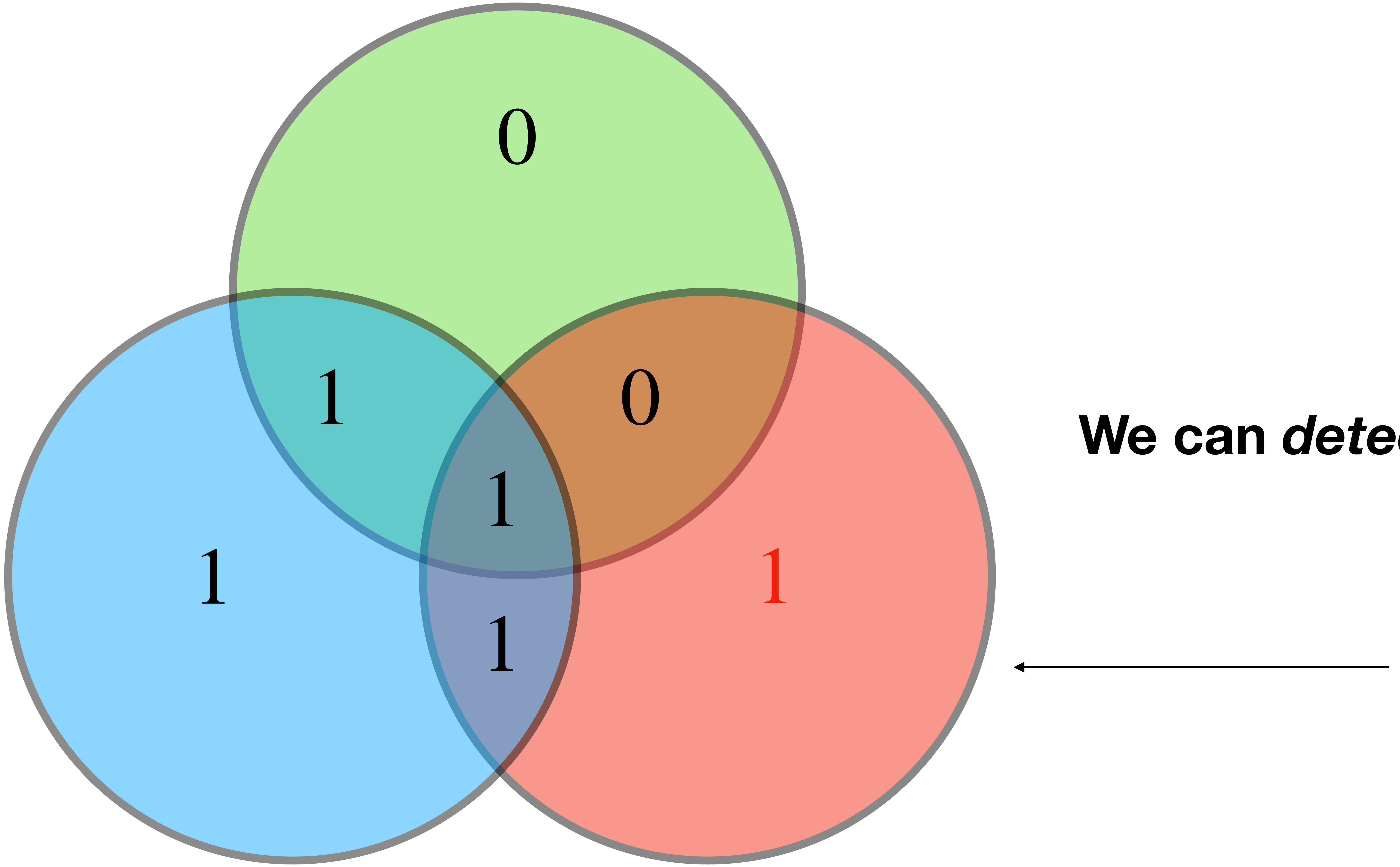


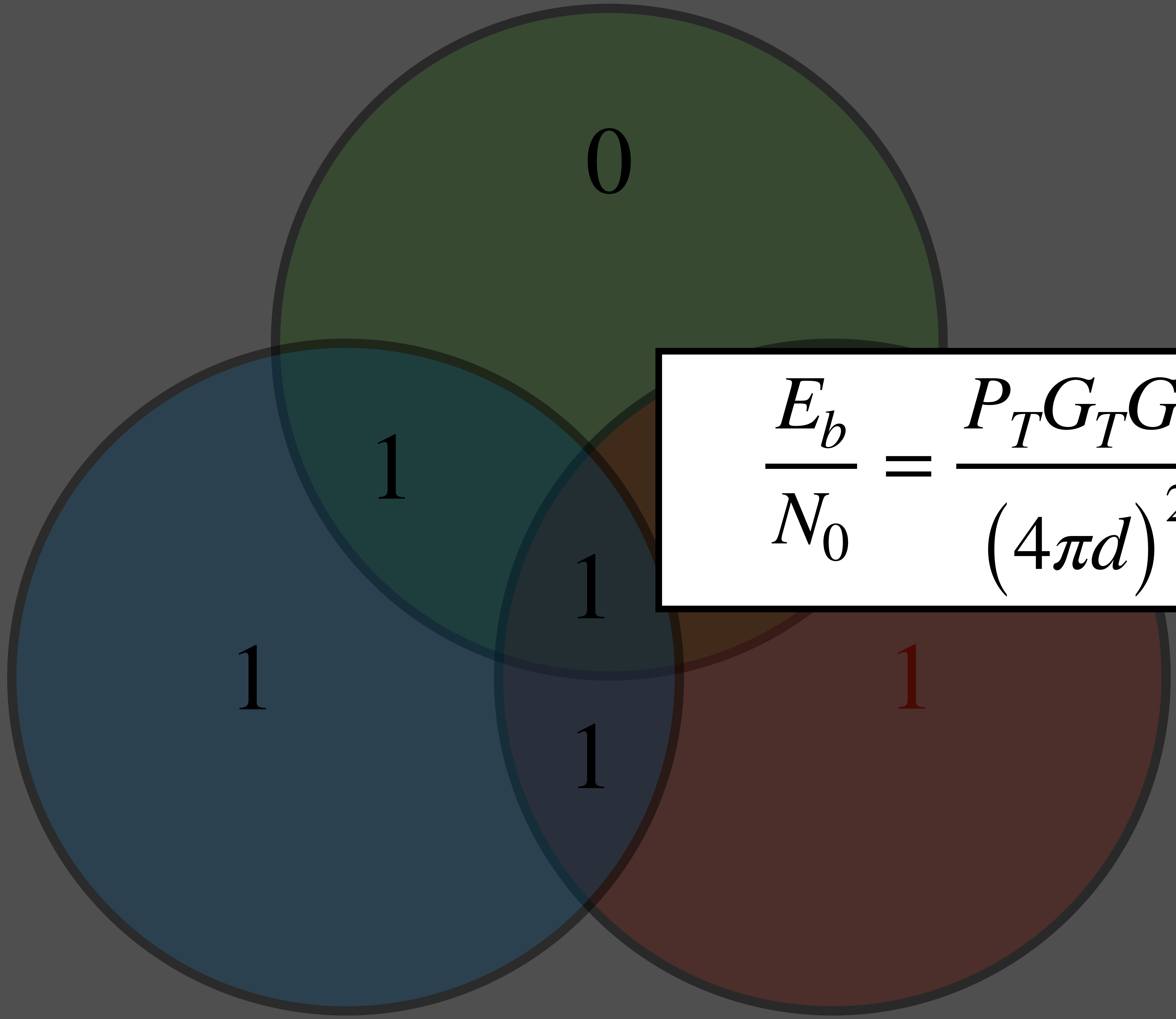
We can *detect* and *correct* errors.

$$[1 \quad 0 \quad 1 \quad 1] \longrightarrow [0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1]$$

This has the effect of decreasing BER **at a fixed** $\dfrac{E_b}{N_0}$

tect **and** ***correct*** **errors.**

$$[1 \quad 0 \quad 1 \quad 1] \longrightarrow [0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1]$$



$$\frac{E_b}{N_0} = \frac{P_T G_T G_R L_a L_l \lambda^2 G_c}{(4\pi d)^2 K_B T_{sys} R_b}$$

tect and *correct* errors.

Matched-filter coding gain

- Matched filtering uses a bit string to represent 1, and an orthogonal bit string to represent 0.
- Signal is found by performing a sliding dot product between known bit strings for 1/0 and the raw data coming off the radio.

$$G_c = 10 \cdot \log_{10}(N)$$

length of matched filter

Reed-Solomon Codes

- Works on symbols (usually 8-bit blocks) rather than bits
- Turns k data symbols into $n > k$ symbols using **polynomials**

Encoding: interpret the message $x = [x_1, x_2, \ldots, x_k]$as coefficients of a polynomial of degree $k - 1$. $p_x(a) = \sum_{i=1}^{k} x_k a^{i-1}$.

Then, evaluate the polynomial at n different points. $p_x(a) = \sum_{i=1}^{k} x_k a^{i-1}$.

Decoding: Based on regression (find the polynomial which goes through the n points.

Being replaced by *turbo codes*, which are a Shannon-limit-approaching code.

Today’s topics:

- Link budget equation
- Modulations
- Bit error rate
- Shannon Limit
- Coding techniques
- **Antennas**
- Examples

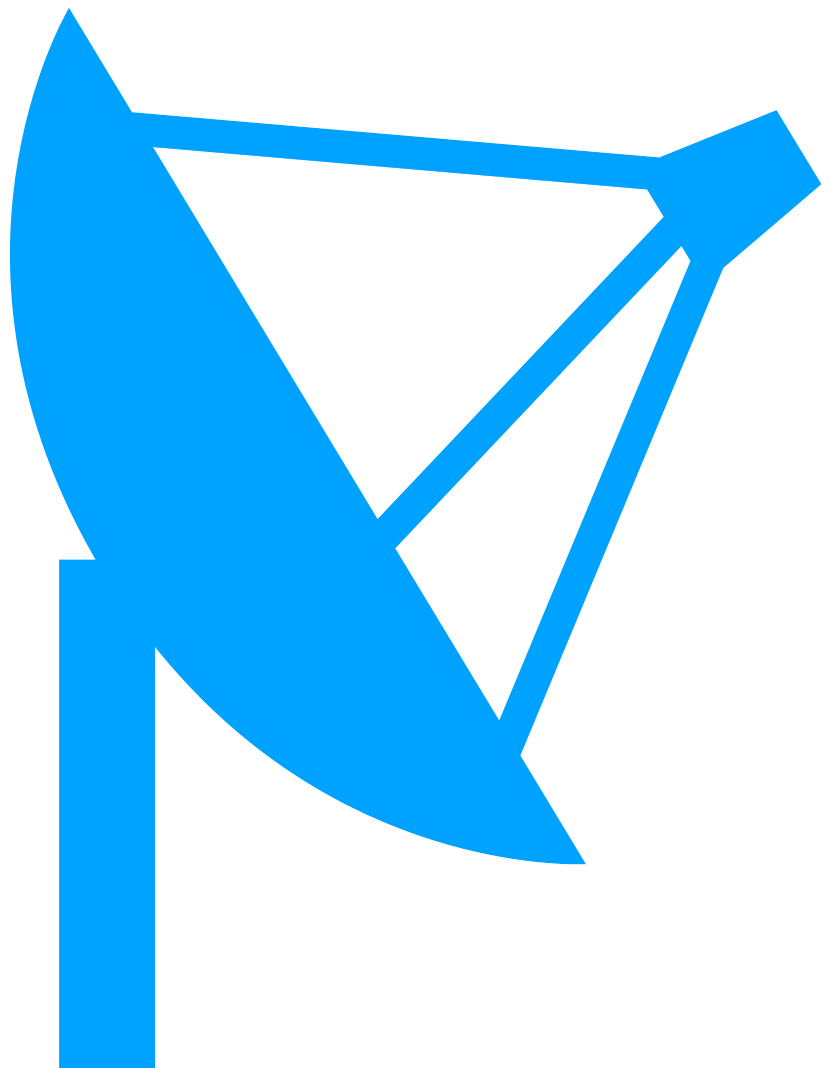
Antennas

Gain for a parabolic antenna of aperture (diameter) D:

$$G = \eta \left(\frac{\pi D}{\lambda} \right)^2$$

dimensionless parameter: “aperture efficiency”

$$0.55 \leq \eta \leq 0.7$$



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- **Examples**

Example

I want to send one 7 megapixel image every minute from Geostationary orbit through a 1 MHz channel at 915 MHz.

1. We'll assume 24-bit color. Each pixel contains RGB information. 8 bits for red, 8 bits for blue, 8 bits for green. So, each picture contains $7{,}000{,}000 \cdot 24 = 168{,}000{,}000$ bits

2. We want to send that many bits every 1 minute. So, our required data rate R_b is

$$\frac{168{,}000{,}000 \text{ bits}}{60 \text{ sec}} = 2.8 \text{ Mbps}$$

3. Given our bandwidth B, we can calculate the required spectral efficiency

$$\eta = \frac{R_b}{B} = \frac{2{,}800{,}000 \text{ bps}}{1{,}000{,}000 \text{ Hz}} = 2.8$$

4. Assume a limit-approaching encoding scheme (e.g. Turbo Codes), and use the Shannon theorem to calculate the minimum $\frac{E_b}{N_0}$ in order to achieve the required spectral efficiency.

$$\frac{E_b}{N_0} \geq \frac{2^\eta - 1}{\eta} = \frac{2^{2.8} - 1}{2.8} = 2.13$$

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$$\frac{E_b}{N_0} \geq \frac{2^{\eta} - 1}{\eta} = \frac{2^{2.8} - 1}{2.8} = 2.13$$

5. Convert to dB: $\left[\frac{E_b}{N_0}\right]_{dB} = 10 \cdot \log_{10}\left(\frac{E_b}{N_0}\right) = 10 \cdot \log_{10}(2.13) = 3.29$ dB

6. Add 3dB of margin: $\frac{E_b}{N_0}_{min} = 6.29$ dB $\longrightarrow$ **Uncertainty in our losses**

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6. Add 3dB of margin: $\frac{E_b}{N_0}_{min} = 6.29$ dB

SMAD 21

7. Use the link budget equation to design your system. Let us assume a 20dB receiver, 5dB of atmospheric/line losses, 10dB of transmit power, a transmission distance of 36,000,000m, a wavelength of 0.32m, a system temperature of 150K. What is the required gain on the transmit antenna?

$$\left[\frac{E_b}{N_0}\right]_{db} = [P_T]_{db} + [G_T]_{db} + [G_R]_{dB} + [L_a]_{dB} + [L_l]_{dB} + 10\log_{10}\left[\left(\frac{\lambda}{4\pi R}\right)^2\right] - 10\log_{10}\left(K_B T_{sys} R_b\right)$$

$$6.29 = 10 + G_T + 20 - 5 + 10\log_{10}\left[\left(\frac{0.32}{4\pi \cdot 36{,}000{,}000}\right)^2\right] - 10\log_{10}\left(1.38 \times 10^{-23} \cdot 150 \cdot 2{,}800{,}000\right)$$

$G_T = 21.9$ **dB**

8. The spacecraft uses a parabolic antenna, how large diameter must it be? Assume an aperture efficiency of 0.6.

Example

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8. The spacecraft uses a parabolic antenna, how large diameter must it be? Assume an aperture efficiency of 0.6.

$$G_T = \eta \left(\frac{\pi D}{\lambda} \right)^2 \longrightarrow D = \sqrt{\frac{G_T}{\eta}} \cdot \lambda \cdot \frac{1}{\pi}$$

Convert G_T back to linear units: $G_T = 10^{\frac{[G_T]_{dB}}{10}} = 10^{2.19} \approx 155$

$$D = \sqrt{\frac{155}{0.6}} \cdot 0.32 \cdot \frac{1}{\pi} = 1.64 \textbf{ meters}$$