Orbital Mechanics and Design Review

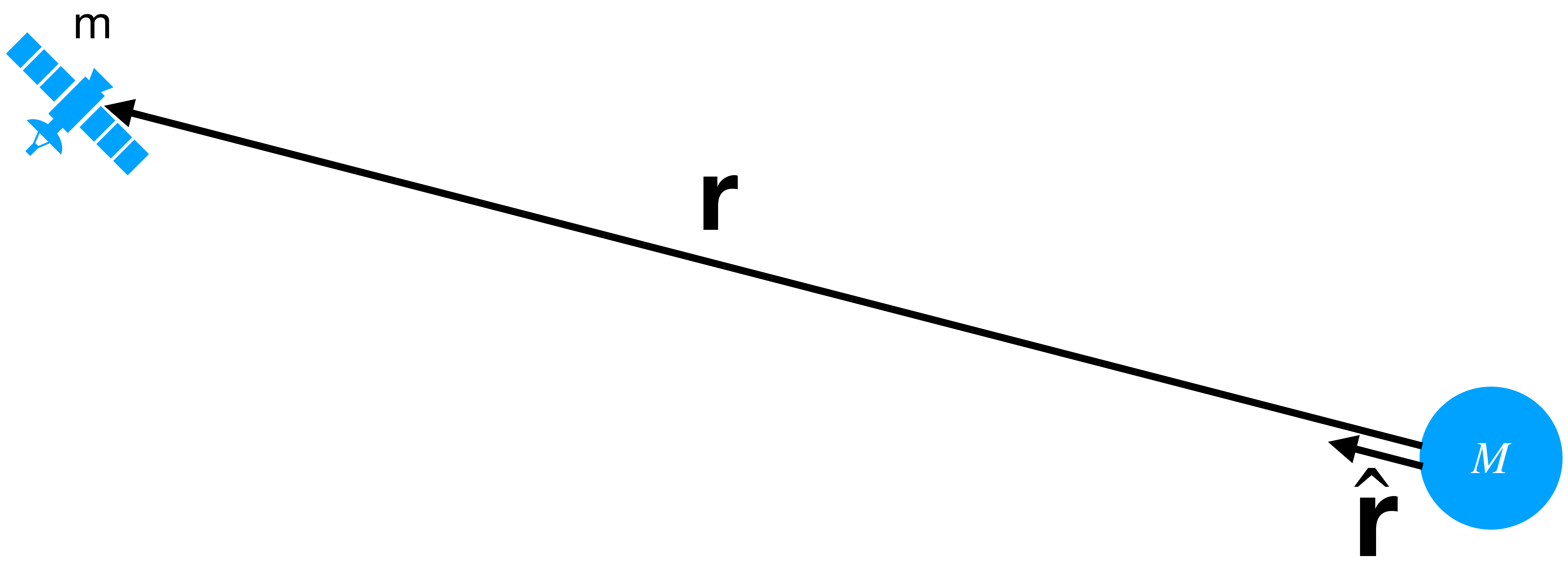
MAE 4160, 4161, 5160

V. Hunter Adams, PhD

Today's topics:

- Brief review of 2-body problem
- Review of Kepler's Laws
- Review of orbital parameters
- Orbital perturbations
- Orbit and constellation design

Two-Body Problem



$$\mathbf{F}_m = m\ddot{\mathbf{r}} = -\frac{GMm}{||\mathbf{r}||^2}$$

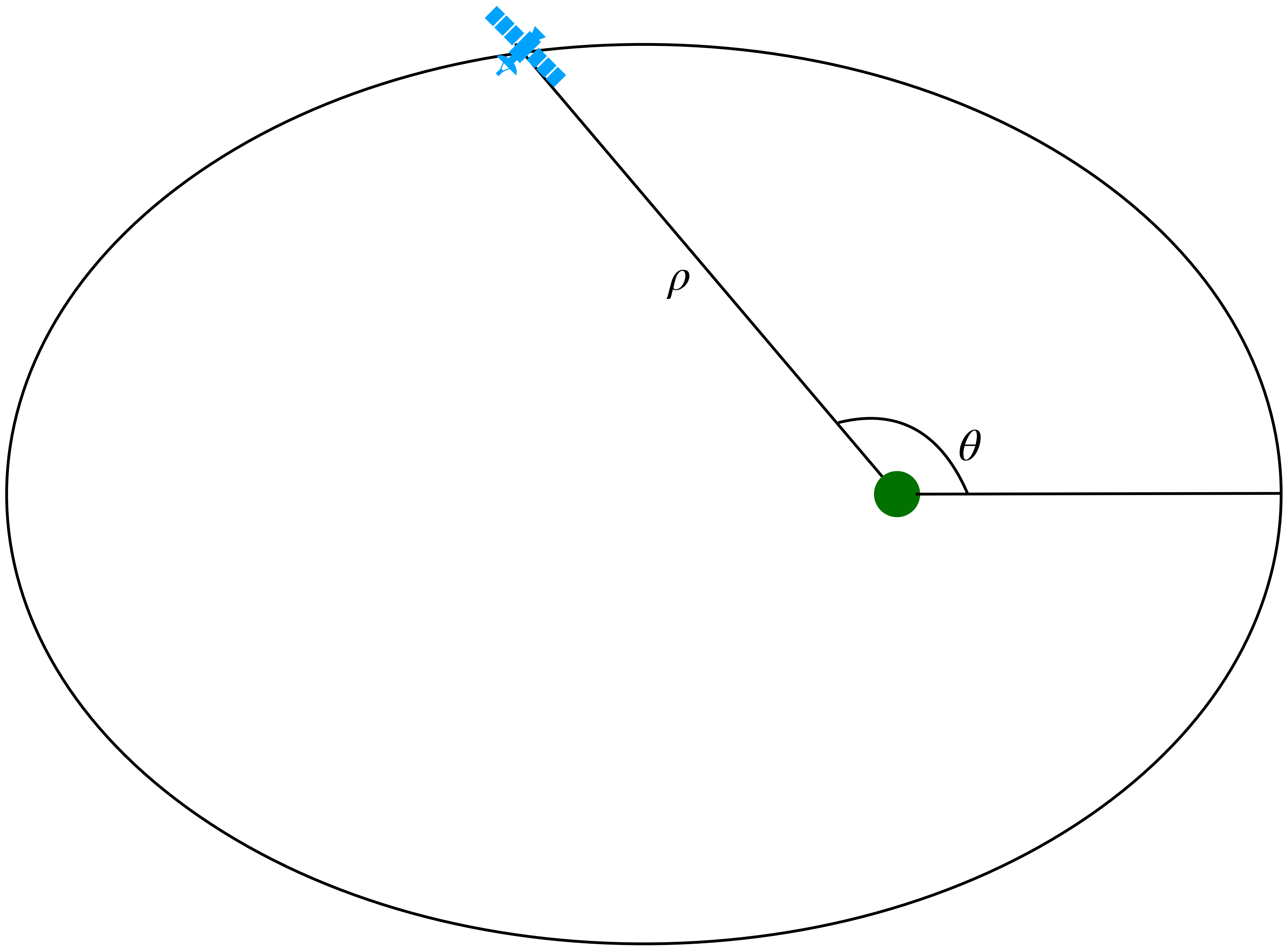
$$\ddot{\mathbf{r}} = -\frac{GM}{||\mathbf{r}||^2}$$

$$\ddot{\rho} = -\frac{GM}{\rho^2} + \rho\dot{\theta}^2$$

$$\ddot{\theta} = -\frac{2\dot{\theta}\dot{\rho}}{\rho}$$

Kepler's Laws

Kepler’s First Law

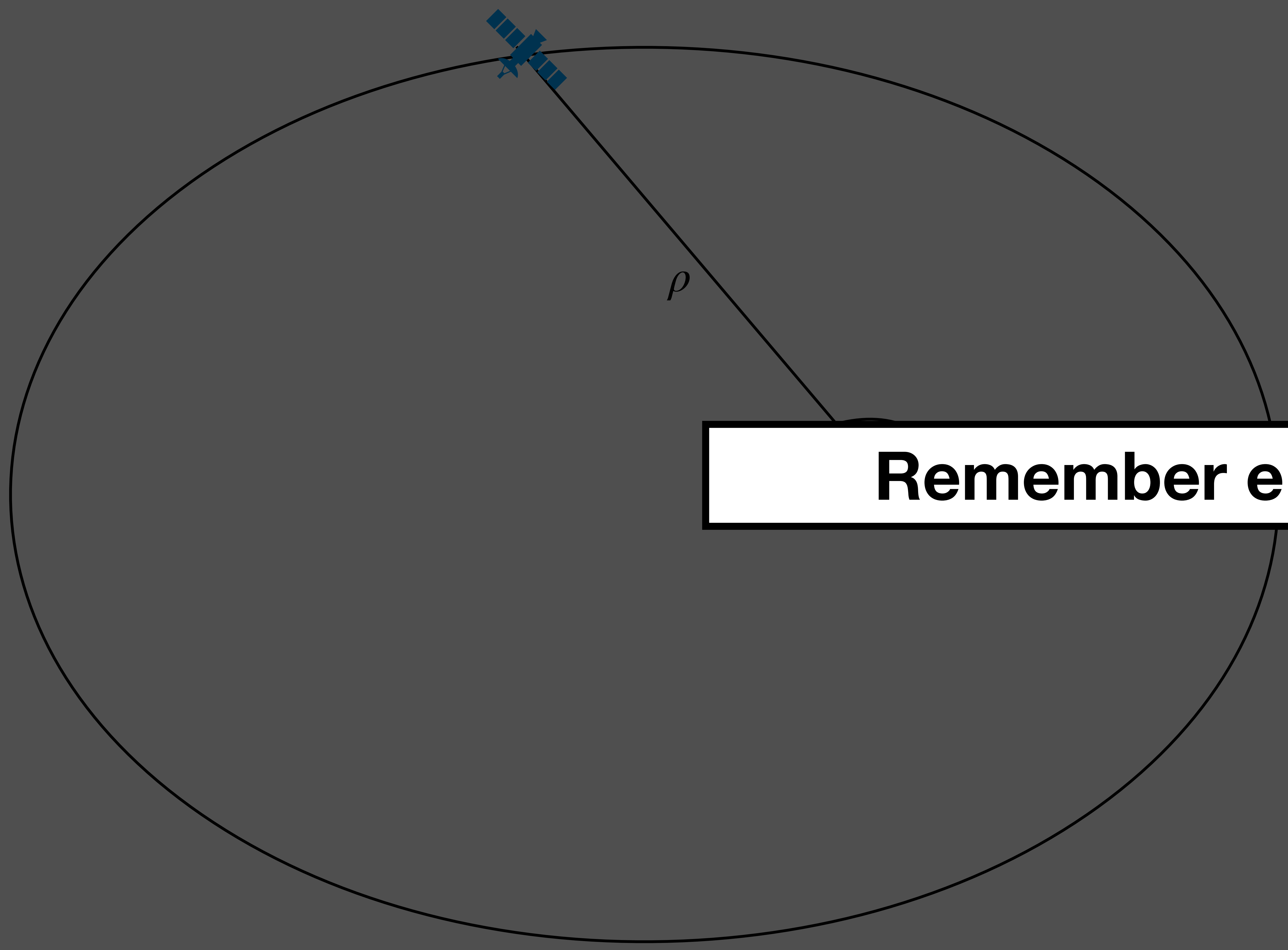


<u>In Words</u>: Orbits are conic sections (circles, ellipses, parabolas, and hyperbolas). They are solutions to the equation below.

$$\rho(\theta) = \frac{a(1-e^2)}{1+e\cos\theta}$$

Where a is the semi major axis, e is the eccentricity, θ is the true anomaly, and ρ is the radial distance from the planet.

Kepler's First Law



Remember ellipses?

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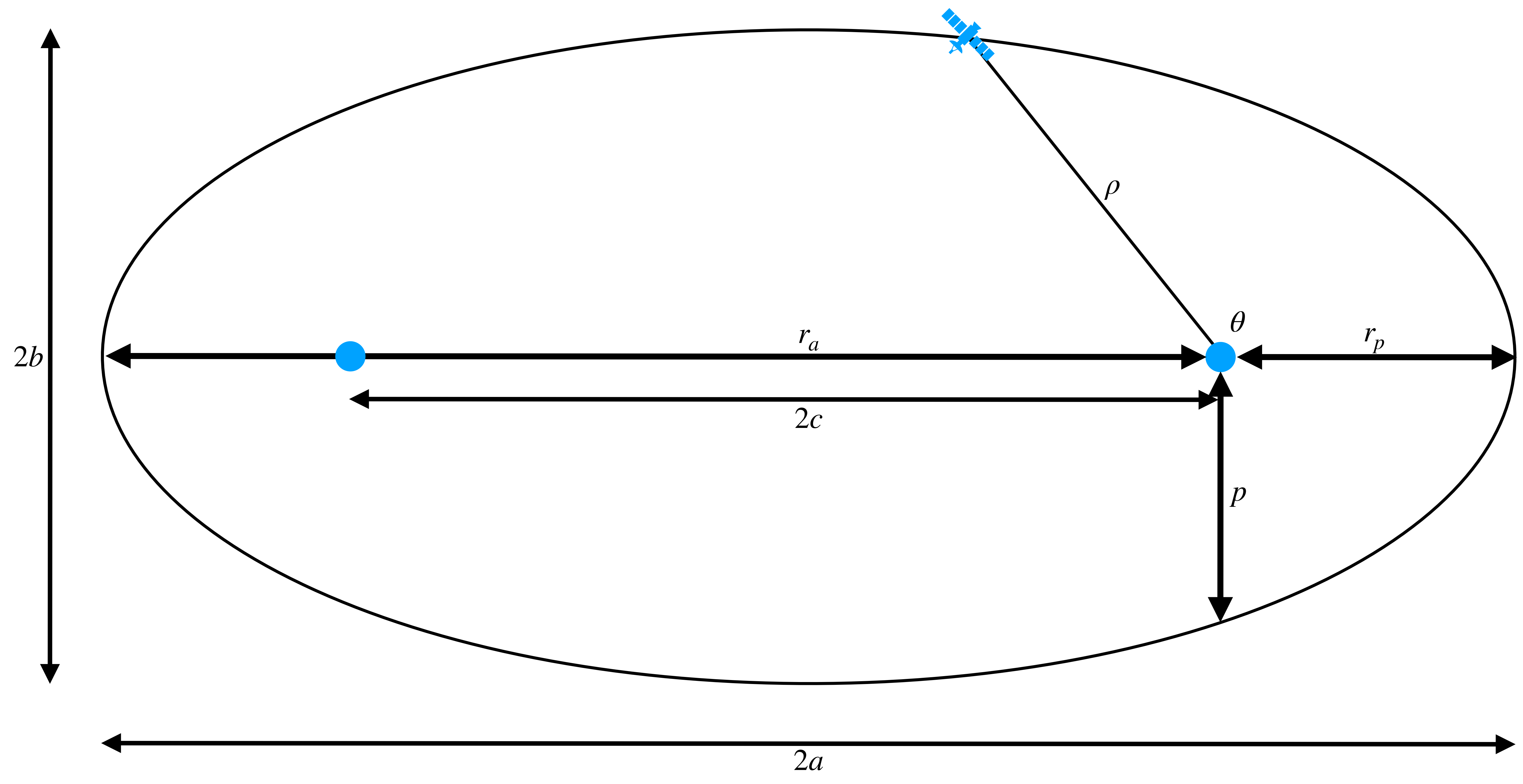
Recalling ellipses: vocabulary and relationships

a: **semi-major axis**
b: **semi-minor axis**
e: **eccentricity**
θ: **true anomaly**
ρ: **radial distance (polar coords)**
p: **semi-latus rectum**
c: **linear eccentricity**
r_a: **apoapsis distance**
r_p: **periapsis distance**
A: **area**



$$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$A = \pi ab$$

$$r_p = a(1 - e)$$

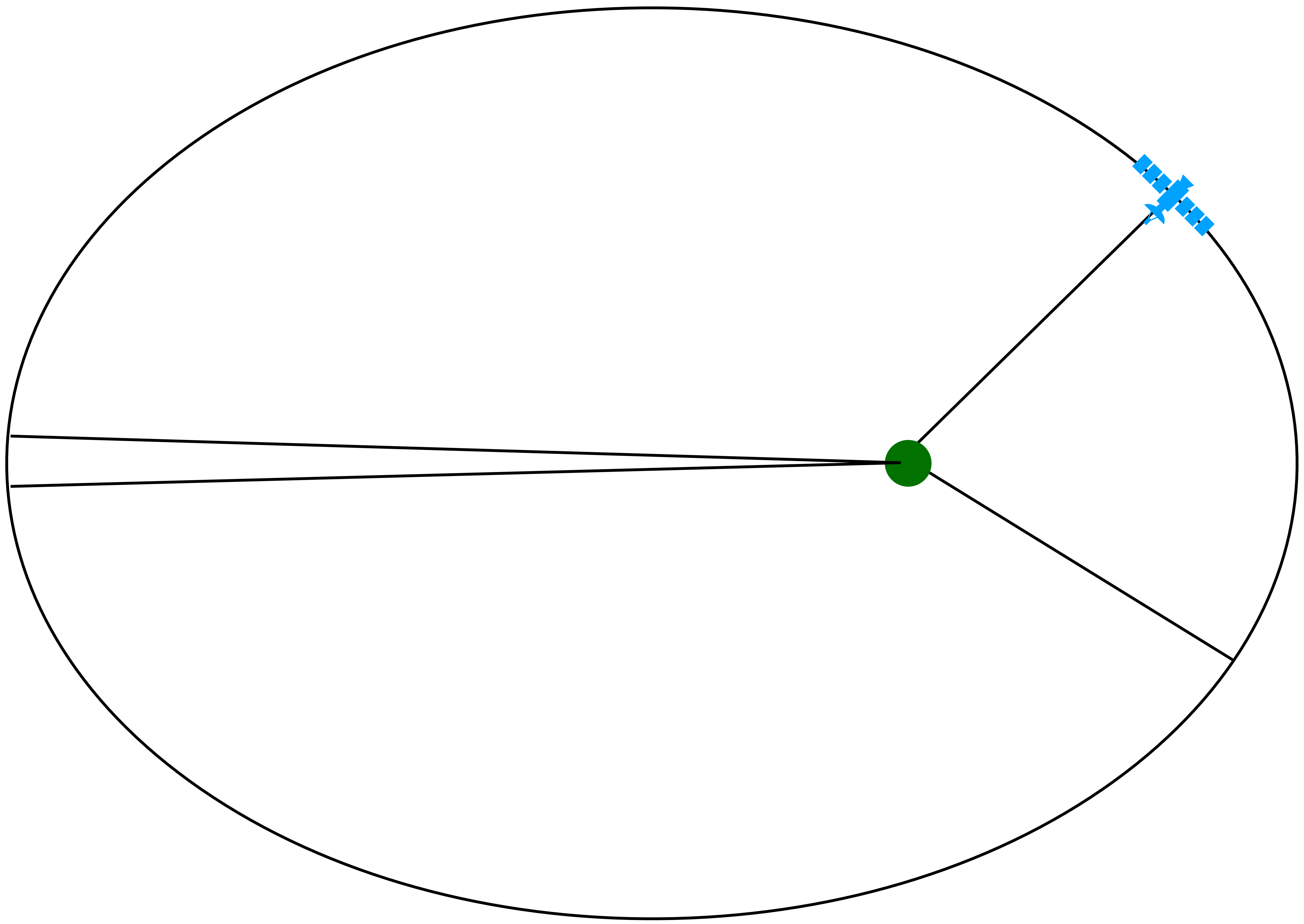
$$r_a = a(1 + e)$$

$$e = \frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}} = \frac{r_p - r_a}{r_p + r_a}$$

$$p = \frac{h^2}{\mu} = a(1 - e^2)$$

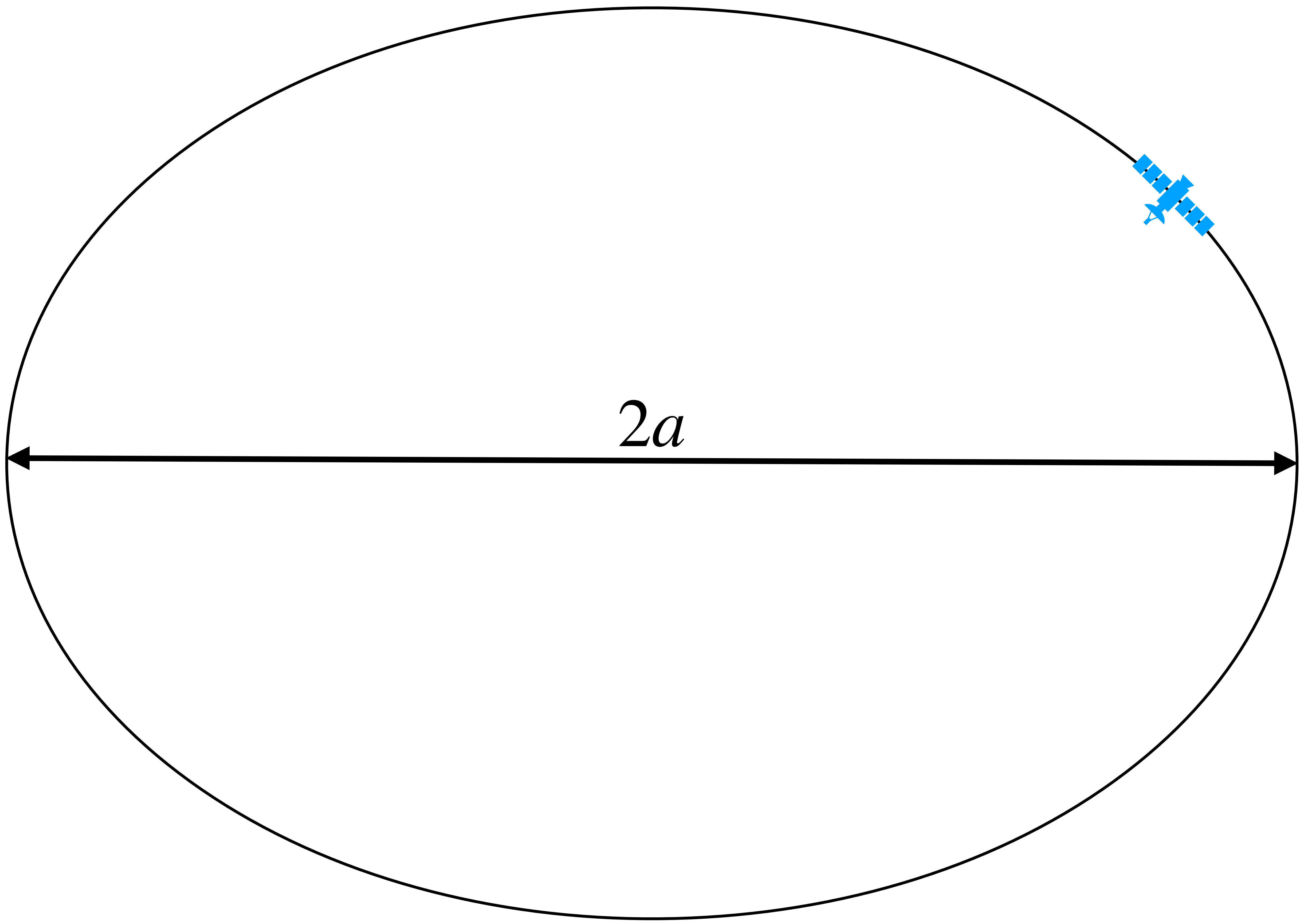
$$\rho = \frac{a(1 - e^2)}{1 + e\cos\theta} = \frac{r_p(1 - e)}{1 + e\cos\theta} = \frac{p}{1 + e\cos\theta}$$

Kepler’s Second Law



<u>In Words</u>: A spacecraft sweeps equal areas in equal times as it orbits a planet.

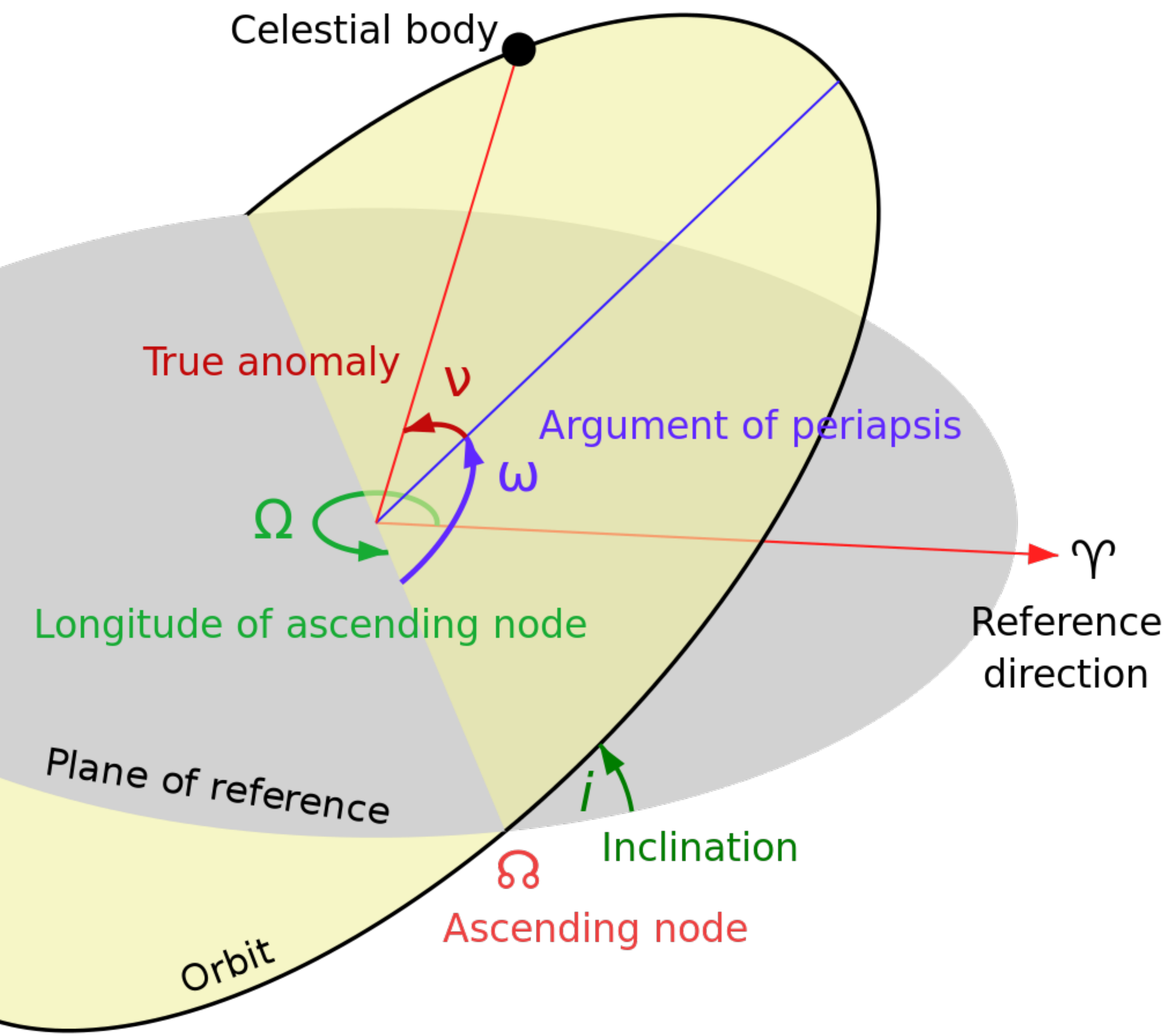
Kepler's Third Law



<u>In Words</u>: The square of the orbital period of a spacecraft is proportional to the cube of the semi-major axis of the orbit.

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

Recalling Orbital Parameters



Size/shape of orbit:

- Semimajor axis, a
- Eccentricity, e

Orientation of orbit:

- Inclination, i
- Longitude of the ascending node, Ω
- Argument of periapsis, ω

Location of satellite in orbit:

- True anomaly, θ

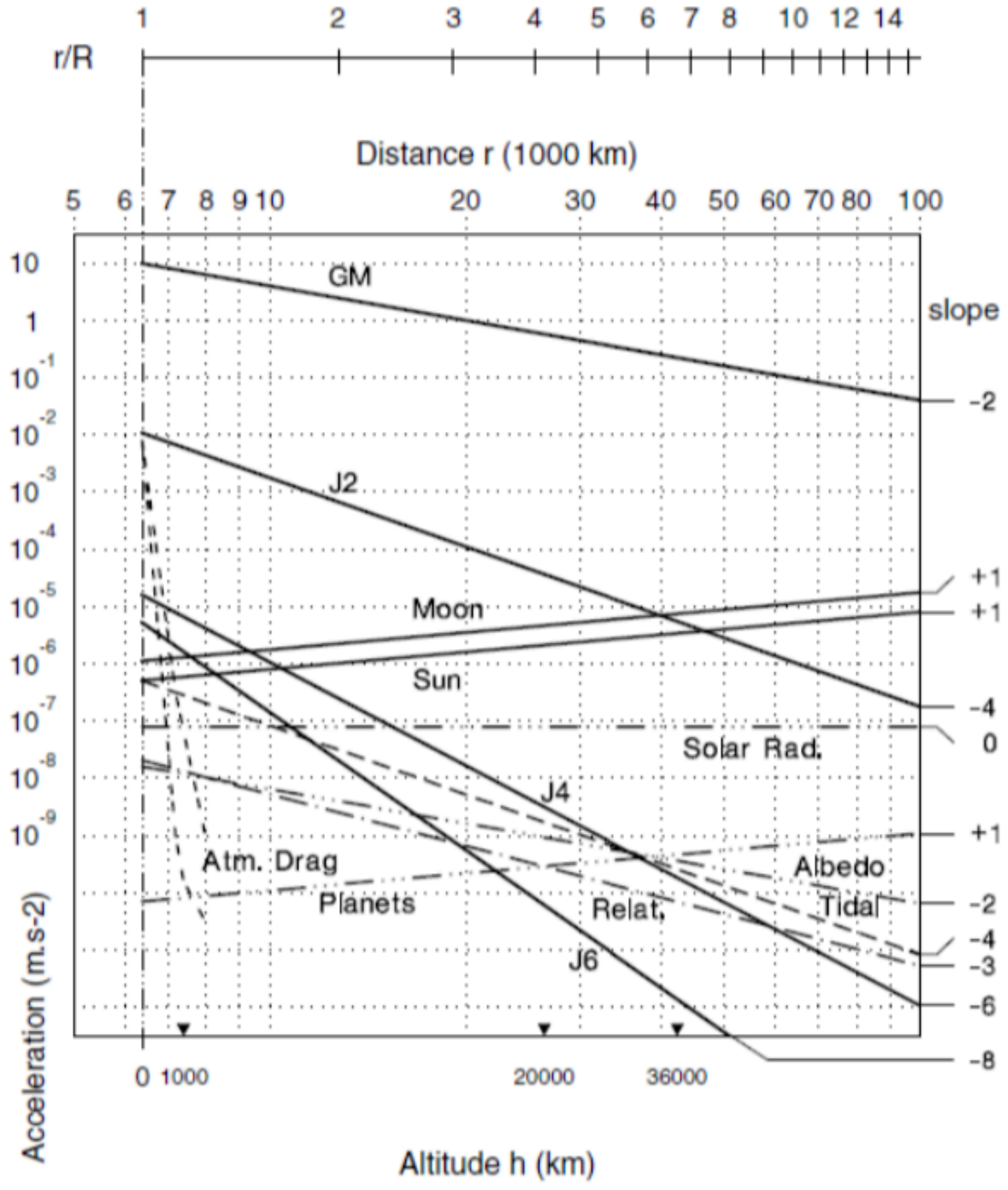


Figure 3.1. Central acceleration and perturbative accelerations as a function of the distance r of the satellite from the centre of the Earth, shown on a log–log scale. Over the ranges considered, the curves can be approximated as straight lines with gradients as noted. The altitudes of the three types of satellite have also been indicated

Orbital Perturbations

Up to ~1000 km, dominant sources of perturbation include:

- Earth oblateness (J2)
- Atmospheric drag

J2 Gravity

Where it comes from: The Earth is not a perfect sphere, nor is it perfectly homogenous in density. It is lumpy and it is fat. This lumpiness affects the Earth's gravitational potential:

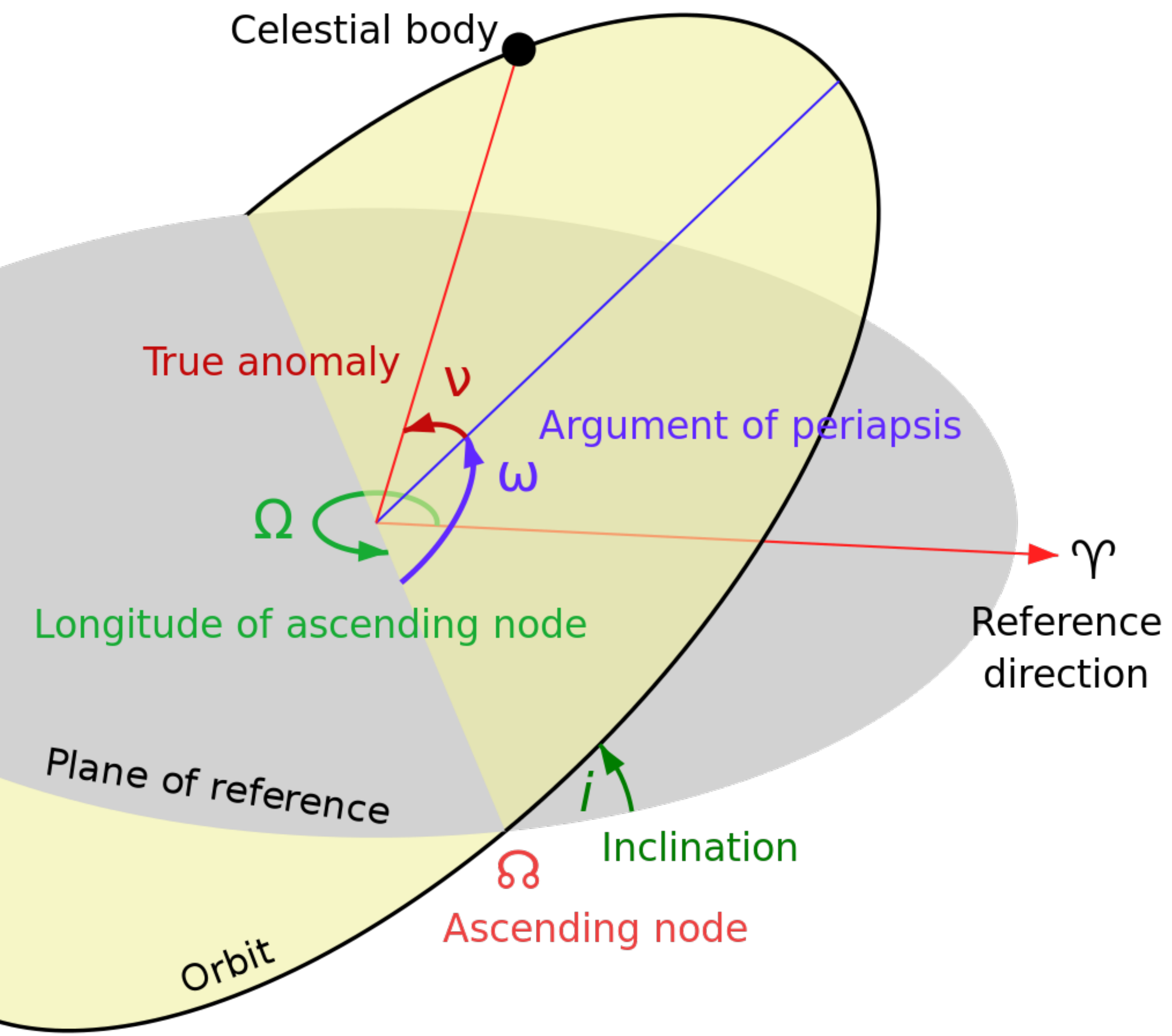
$$u = \underbrace{-\frac{\mu}{r}}_{\text{Point-mass}} + \underbrace{\sum_{n=2}^{N_z} \frac{J_n P_n^0(\sin\theta)}{r^{n+1}}}_{\text{Zonal terms}} + \underbrace{\sum_{n=2}^{N_t}\sum_{m=1}^{n} \frac{P_n^m(\sin\theta)(C_n^m \cos m\varphi + S_n^m \sin m\varphi)}{r^{n+1}}}_{\text{Tesseral terms}}$$

The first term in the above expansion (the "J2" terms) have the following effects on the orbital parameters:

$$\dot{\Omega} = -\frac{3}{2(1-e^2)^2} n J_2 \left(\frac{R_E}{a}\right)^2 \cos i \longrightarrow \textbf{Nodal precession}$$

$$\dot{\omega} = \frac{3}{4(1-e^2)^2} n J_2 \left(\frac{R_e}{a}\right)^2 \left(5\cos^2 i - 1\right) \longrightarrow \textbf{Apsidal precession}$$

Recalling Orbital Parameters



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Orientation of orbit:

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Location of satellite in orbit:

- True anomaly, θ

Atmospheric Drag

$$a_d(r, V) = -\frac{1}{2}\rho(r)V(r)^2 \cdot \frac{C_D A}{m}$$

See SMAD for equations that model the changes in semi major axis/eccentricity per orbit.

Factors into your Delta-V budget!

Atmospheric density is hard to model - more on that in a later lecture.

Orbit and constellation design

Cool idea: Let’s provide persistent, high-definition television coverage to the USA

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More specifically: We require persistent transmission of [frequency range] to [latitude/longitude range that includes USA] at [bandwidth]

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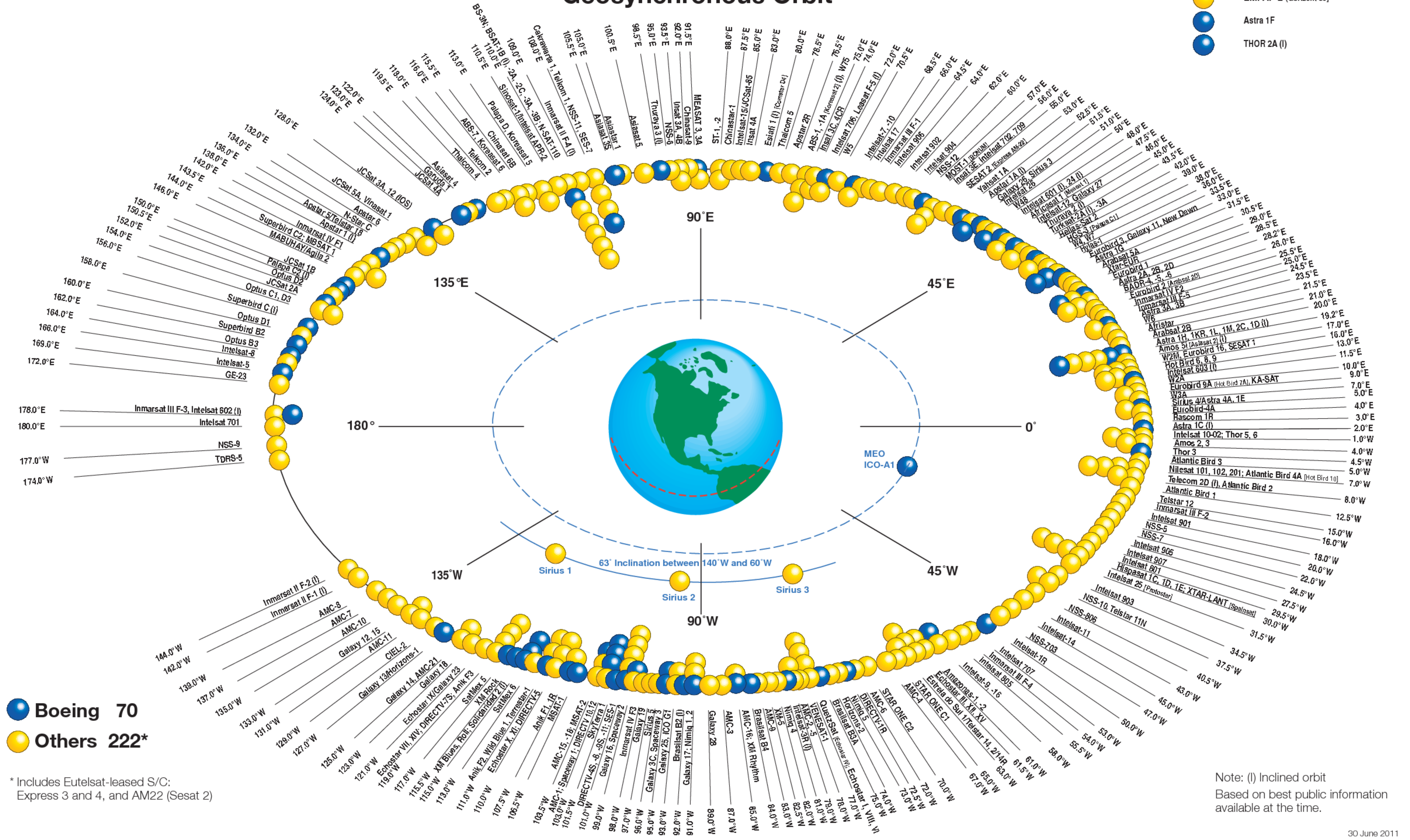
The answer: Geostationary orbit

Commercial Communications Satellites
Geosynchronous Orbit



* Includes Eutelsat-leased S/C: Express 3 and 4, and AM22 (Sesat 2)

Note: (I) Inclined orbit

Based on best public information available at the time.

30 June 2011

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(can’t use the same solution as before - why?)

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More specifically: We require persistent transmission of [frequency range] to [latitude/longitude range that includes Russia] at [bandwidth]

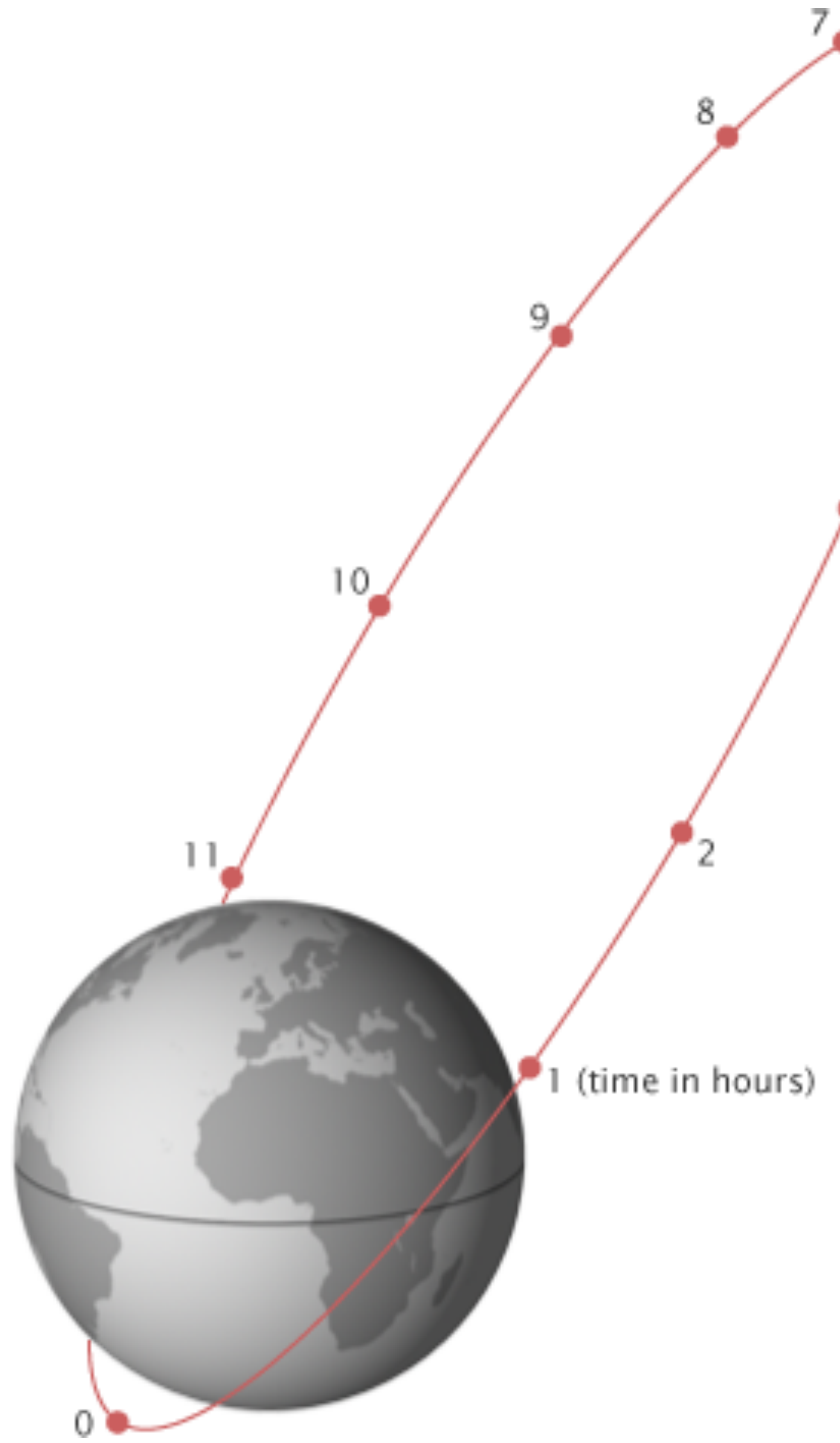
(can’t use the same solution as before - why?)

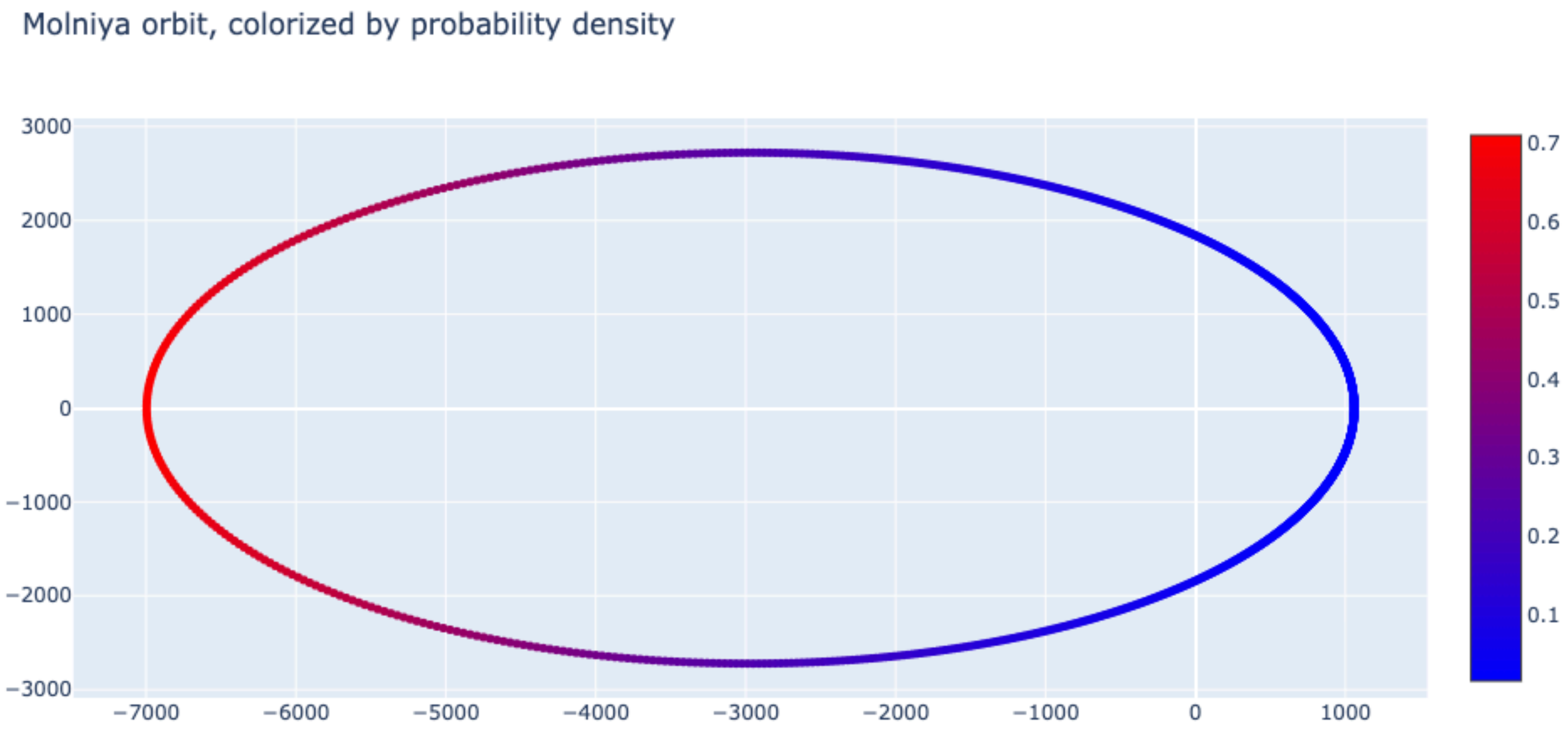
The answer: Molniya orbits

Goals (for this example, and all others):

1. Design a constellation that meets requirements
2. Design the constellation such that perturbations to not warp the constellation over time in ways that make it not longer meet requirements.

$$p(\theta) = \frac{(1-e)^{3/2}}{2\pi \left(\frac{1}{e+1}\right)^{3/2} (e\cos(\theta)+1)^2}$$

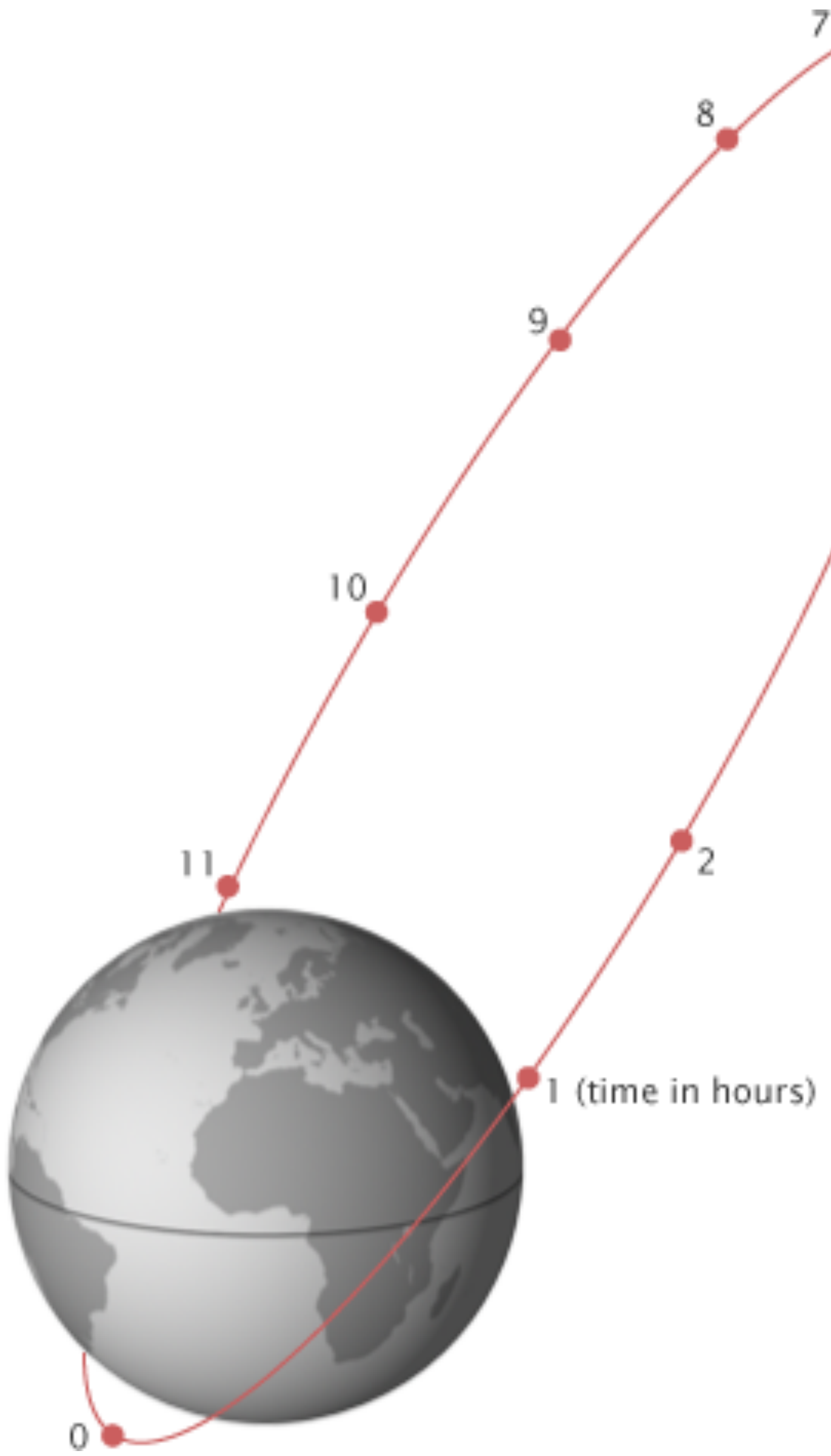




We need to deal with apsidal precession and nodal precession.

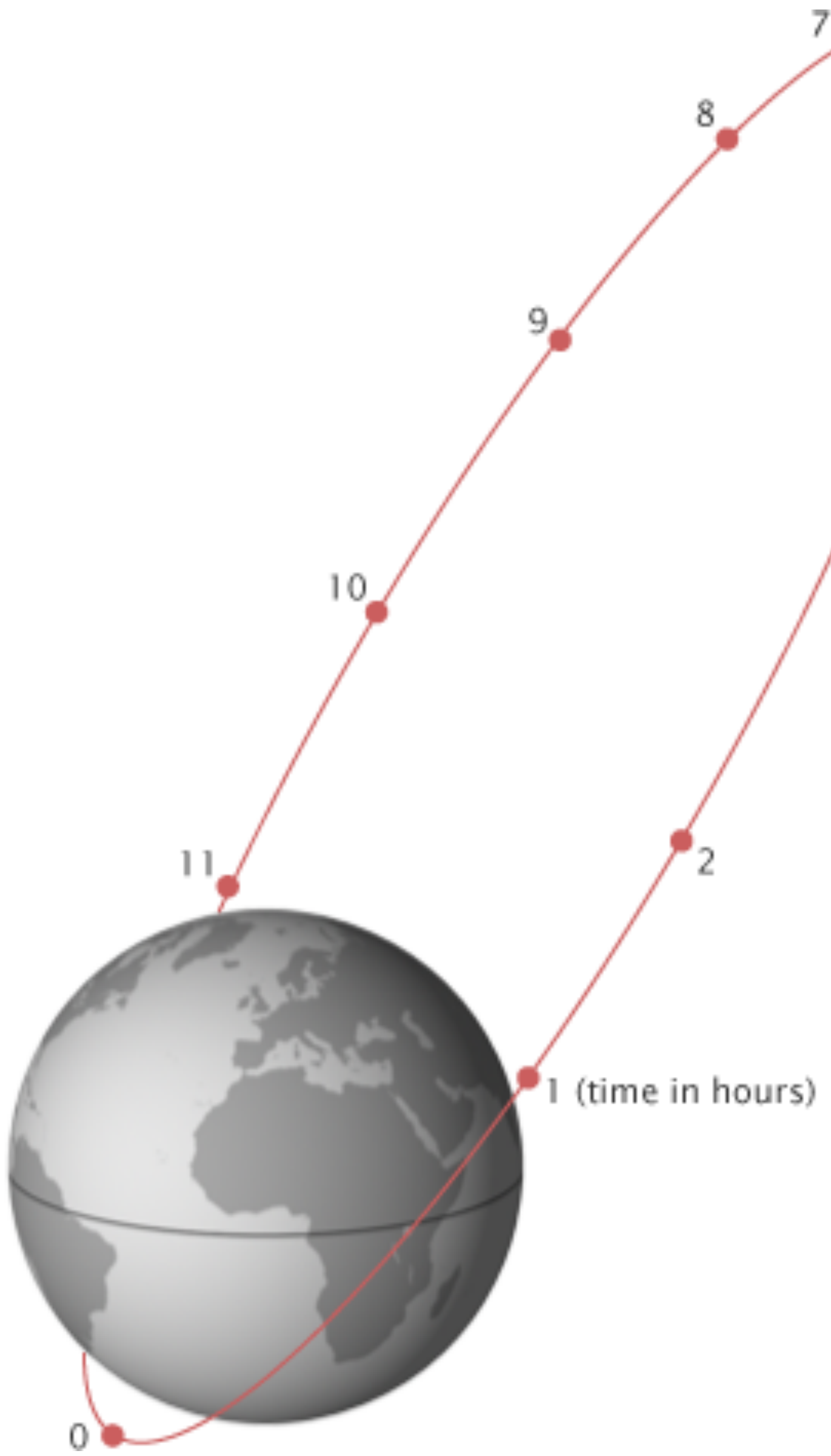
Apsidal precession

$$\dot{\omega} = \frac{3}{2} \cdot nJ_2 \cdot \left(\frac{R_E}{a}\right)^2 \frac{2 - 2.5\sin^2 i}{\left(1 - e^2\right)^2}$$

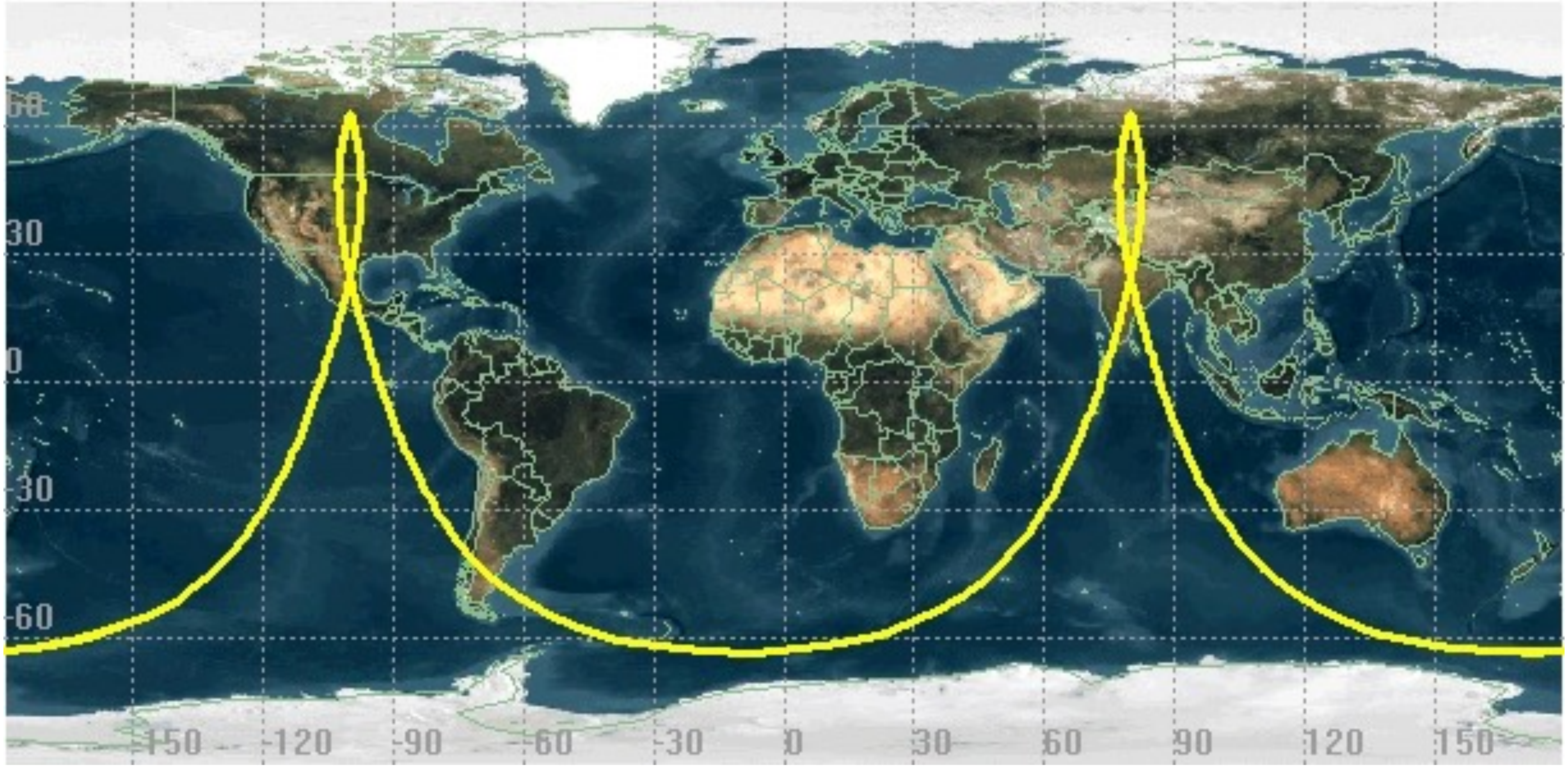


Nodal precession

$$\dot{\Omega} = -\frac{3}{2} \cdot nJ_2 \cdot \left(\frac{R_E}{a}\right)^2 \frac{\cos i}{\left(1 - e^2\right)^2}$$



Think about this visually using the ground-track.


60
30
0
-30
-60
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-120
-90
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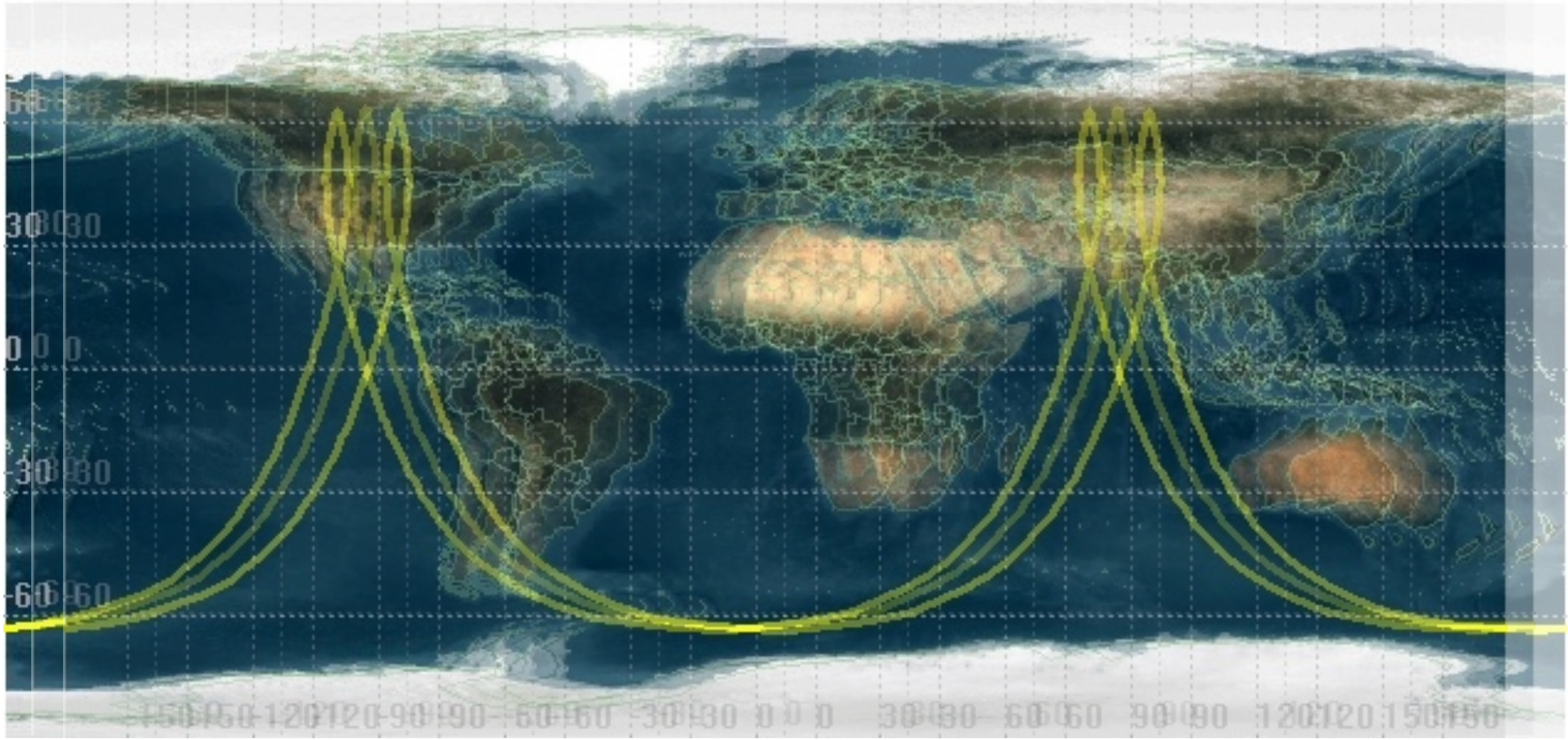
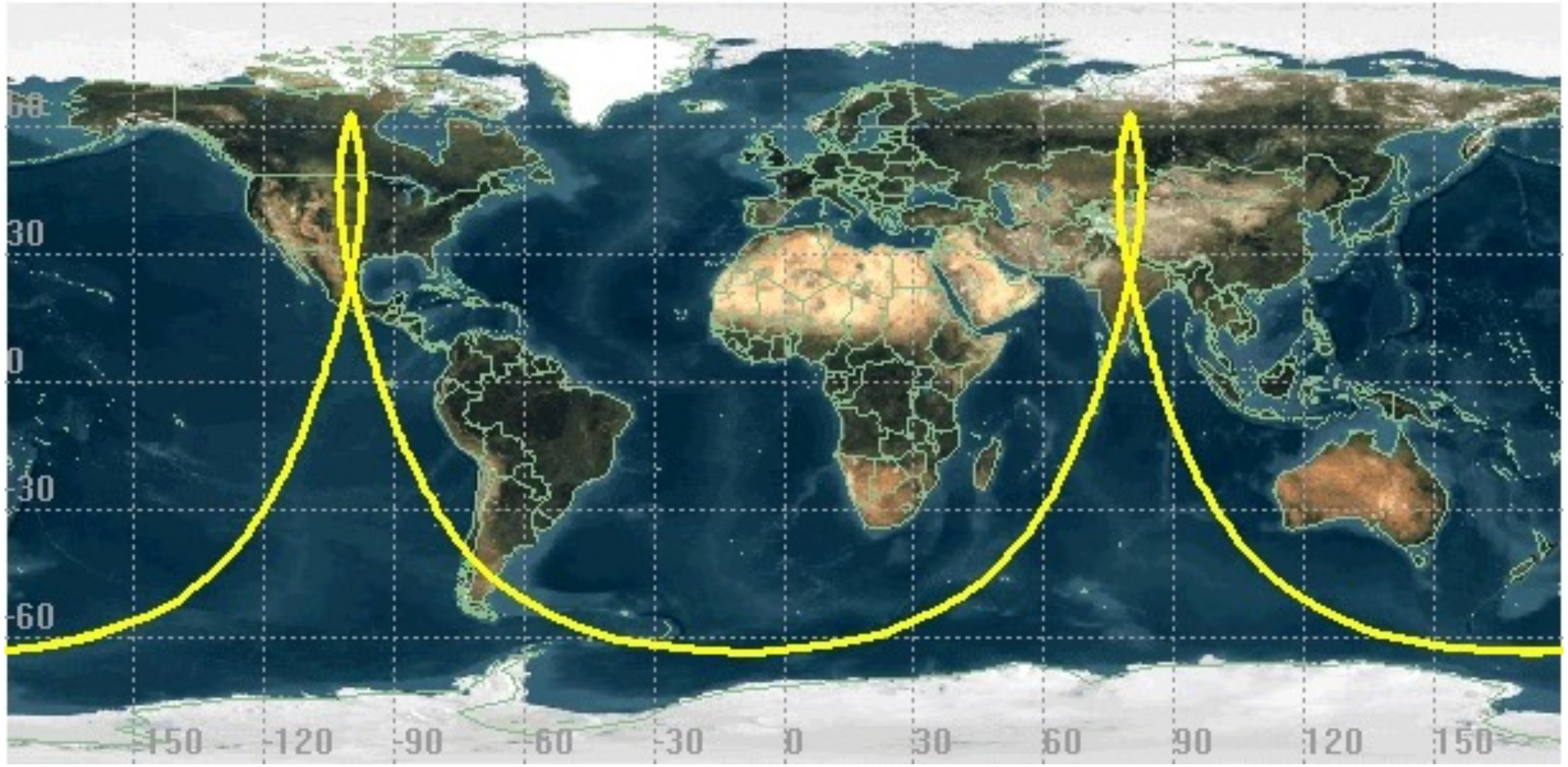
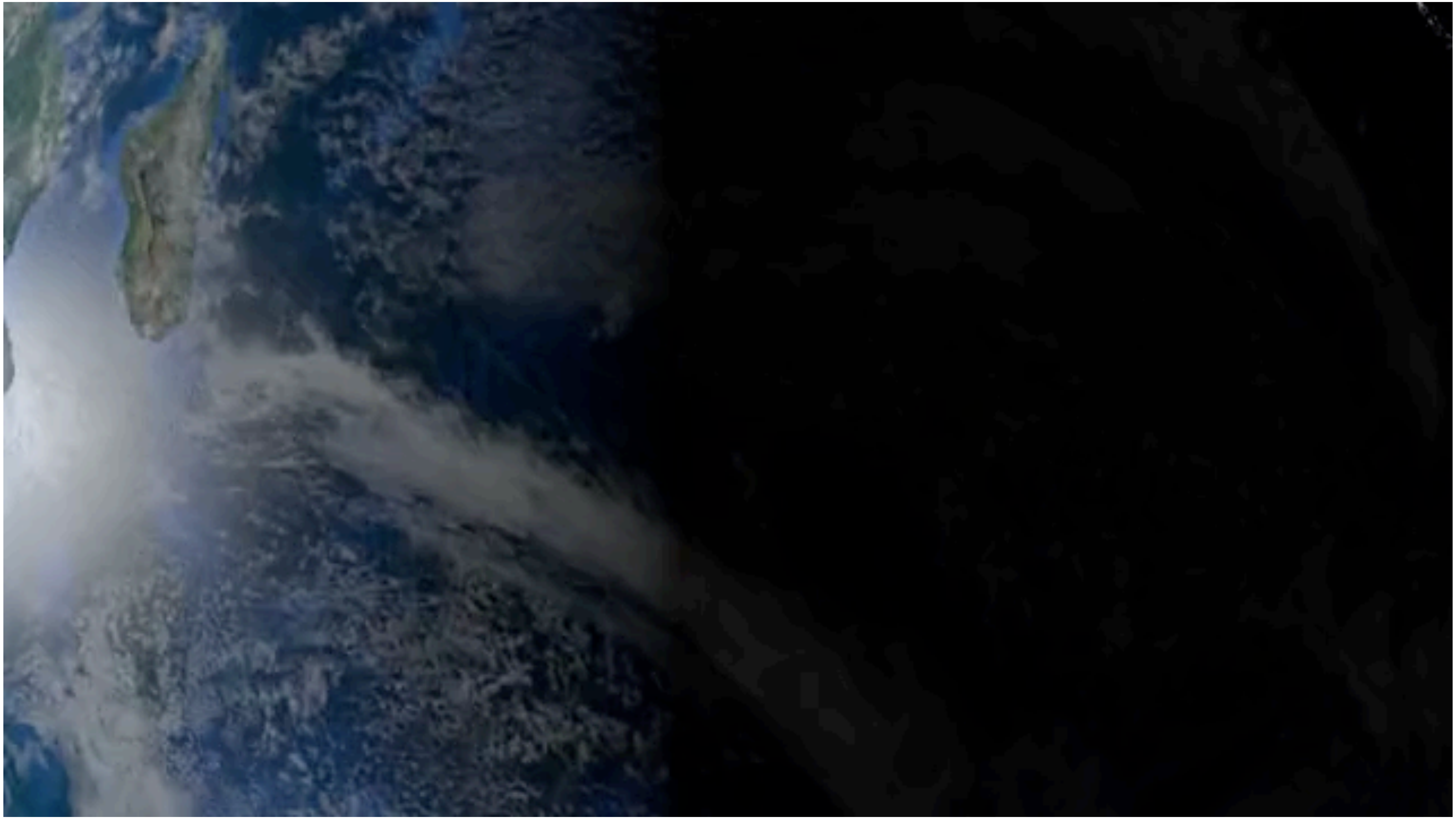


(ignore the fact that the Earth is blurry)

Results

1. Inclination of 63.4 degrees (to freeze latitude of apogee)
2. Perigee at 600 km (as low as possible, so that our apogee can be as high as possible)
3. Apogee at ~39,700 km (for an orbital period which is exactly 1/2 sidereal days - giving us a non-drifting ground track)
4. Eccentricity of 0.74 (as calculated from apogee/perigee)
5. Argument of perigee at 270, so that apogee is at northern-most point.





But! This doesn't give persistent coverage.

~8 hours/day

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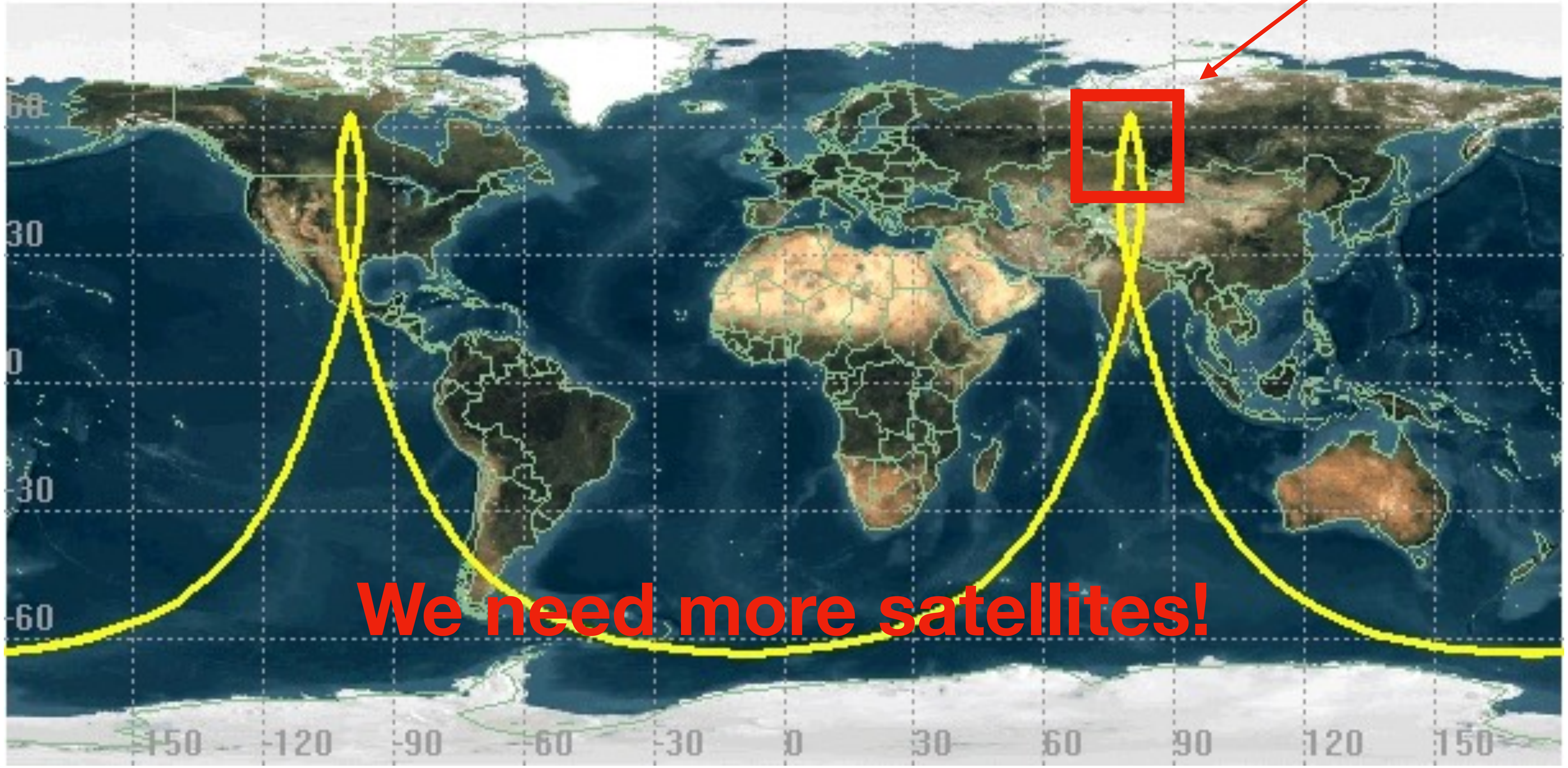
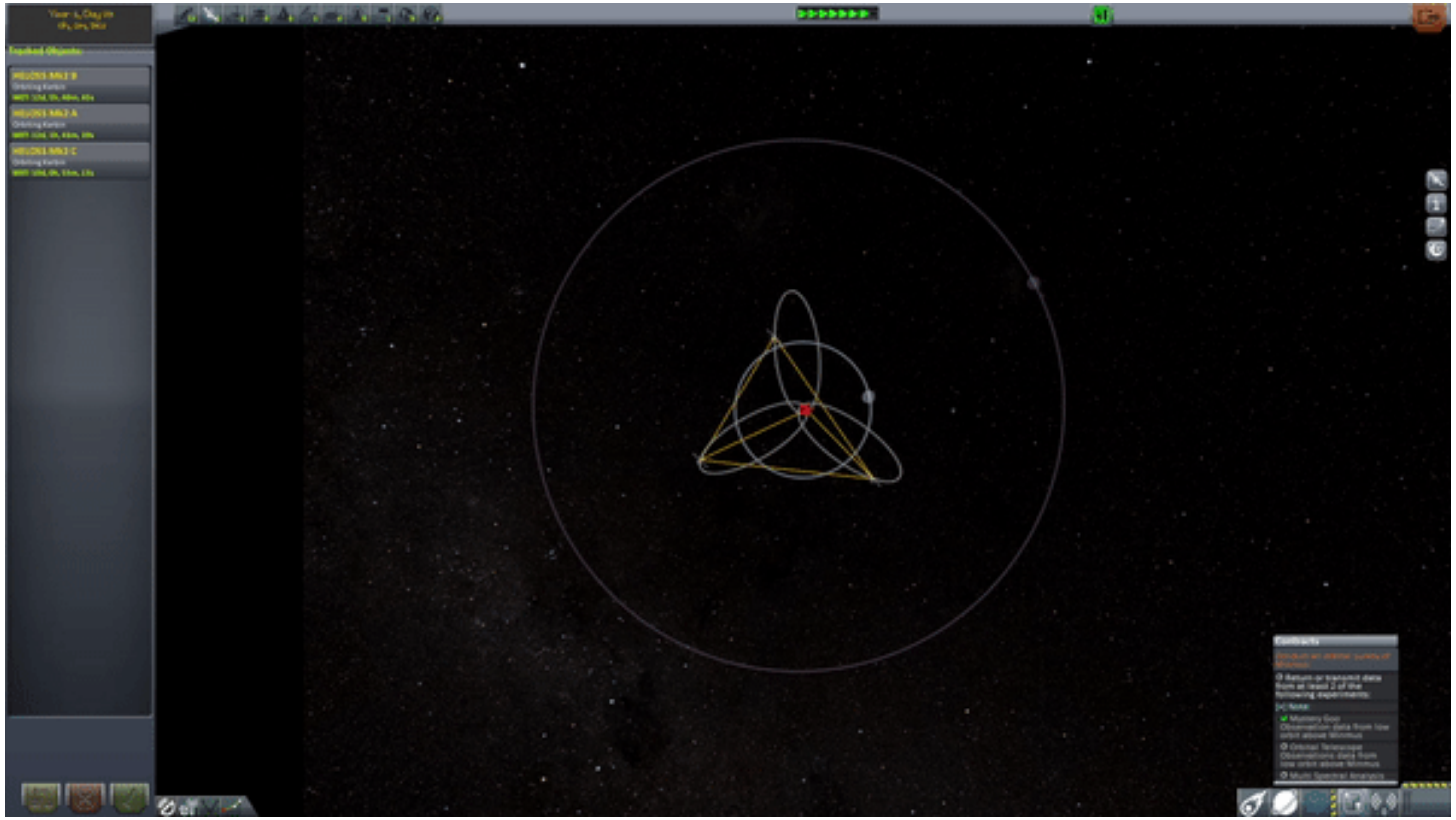


3 spacecraft, separated in longitude of ascending node by 120 degrees, phased such that their apogees occur ~8 hours apart.



Think about this visually using the ground tracks also.

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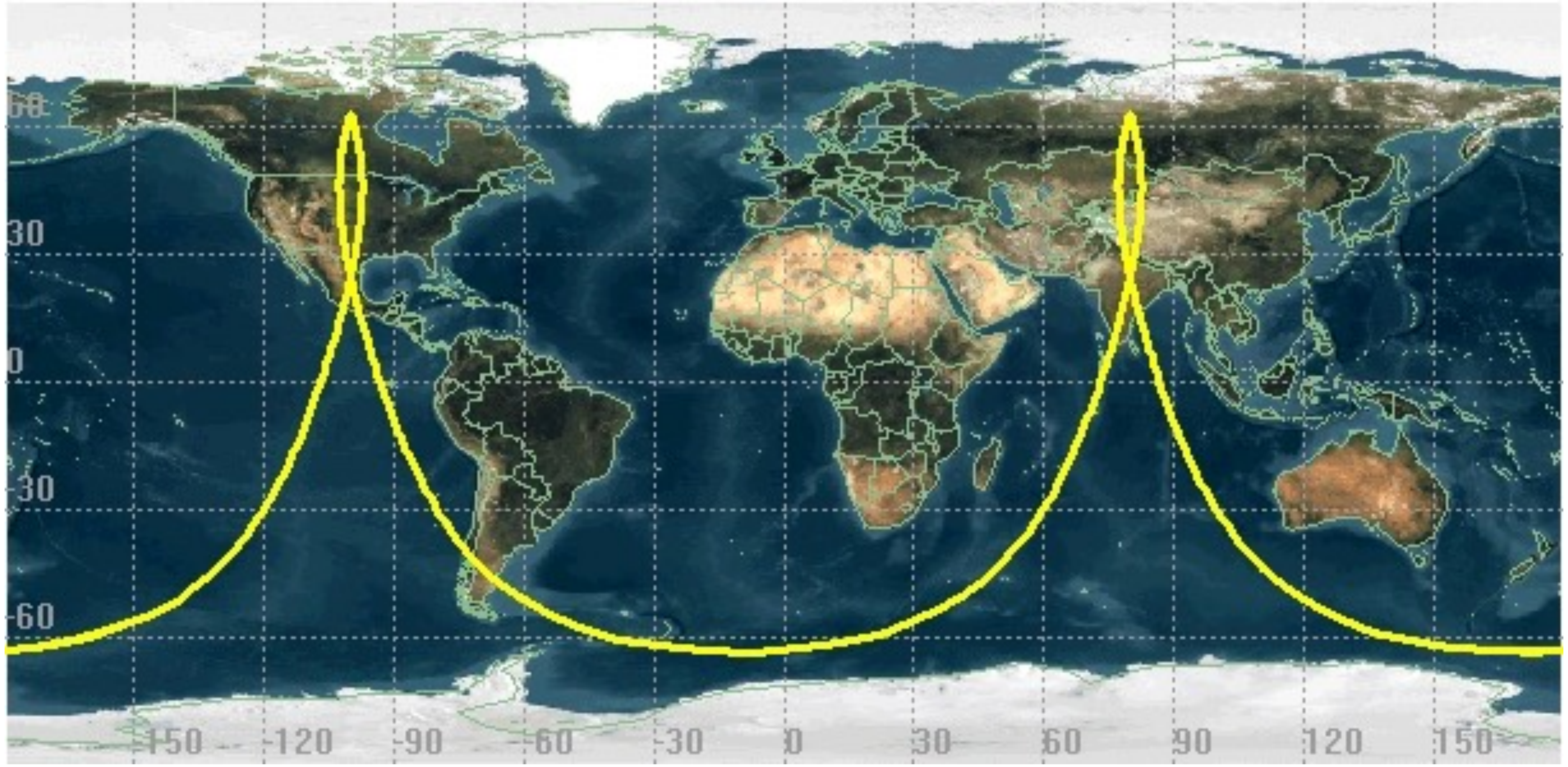
6. 3 spacecraft, on the orbit described above, separated by 120 deg in lon. of ascending node and phased for ~8 hours between apogees



Tundra orbit (by the way)

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More specifically (skipping over a lot of reasoning): Design a constellation such that at least 4 satellites are at least 15 degrees above the horizon at all times from any location on the surface of the Earth.

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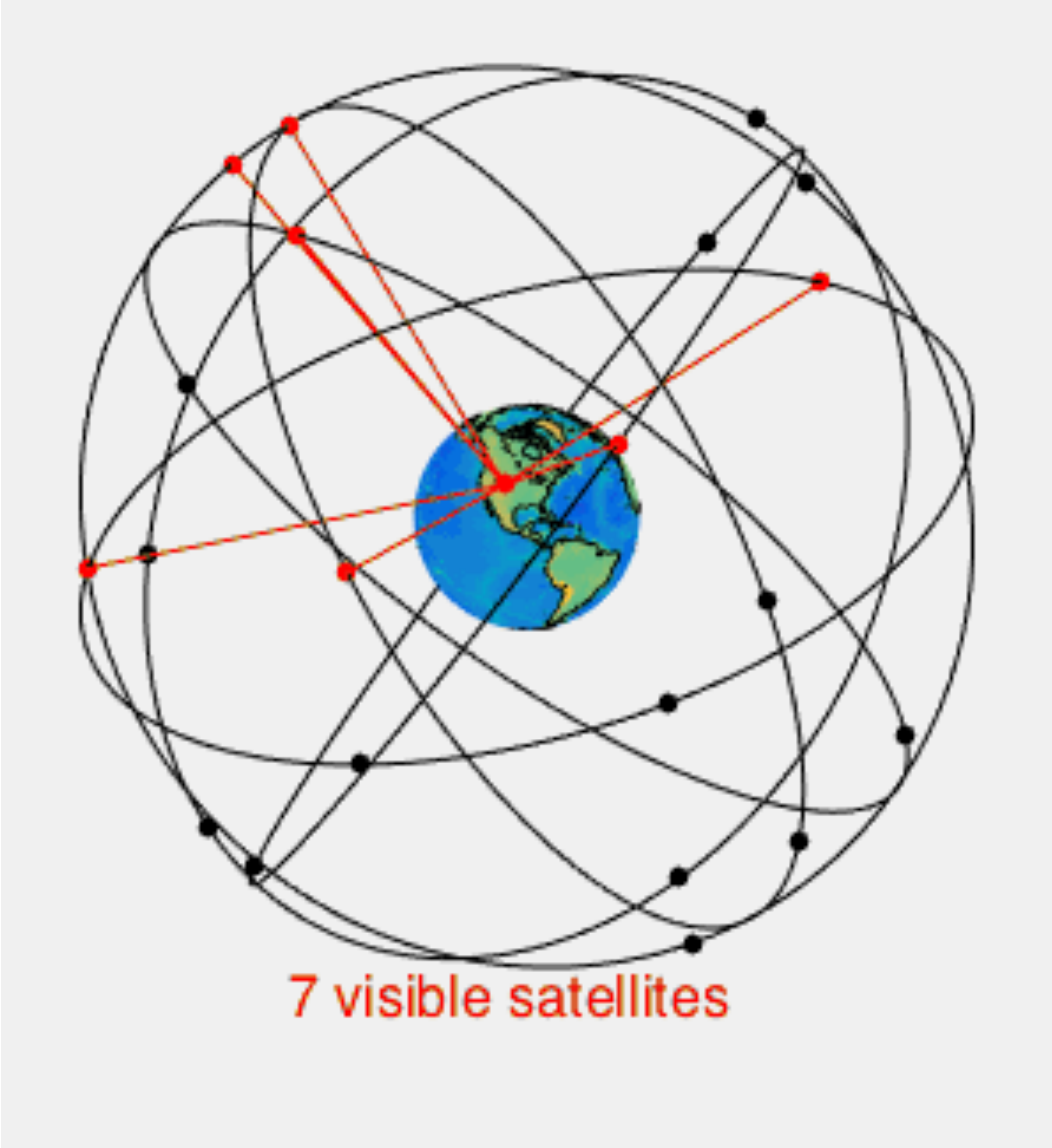
Answer: There’s more than one. I’m going to cheat. I’ll show you the answer and show you how it meets requirements, rather than come up with the answer based on requirements.

Remember the goals.

1. Design a constellation that meets requirements
2. Design the constellation such that perturbations to not warp the constellation over time in ways that make it not longer meet requirements.

Do you see how symmetry might be an important element of this constellation?

GPS Constellation



Properties:

- **6 orbital planes**
- **55 degree inclination for each plane**
- **Separation of 60 degrees in the RAAN for each plane**
- **Circular orbits**
- **Semi-synchronous (2 orbits = 1 Earth rotation)**
- **24 satellites (plus spares)**
- **Non-uniform distribution of satellites in each plane**

Let's consider how each of these properties helps us meet and maintain requirements.

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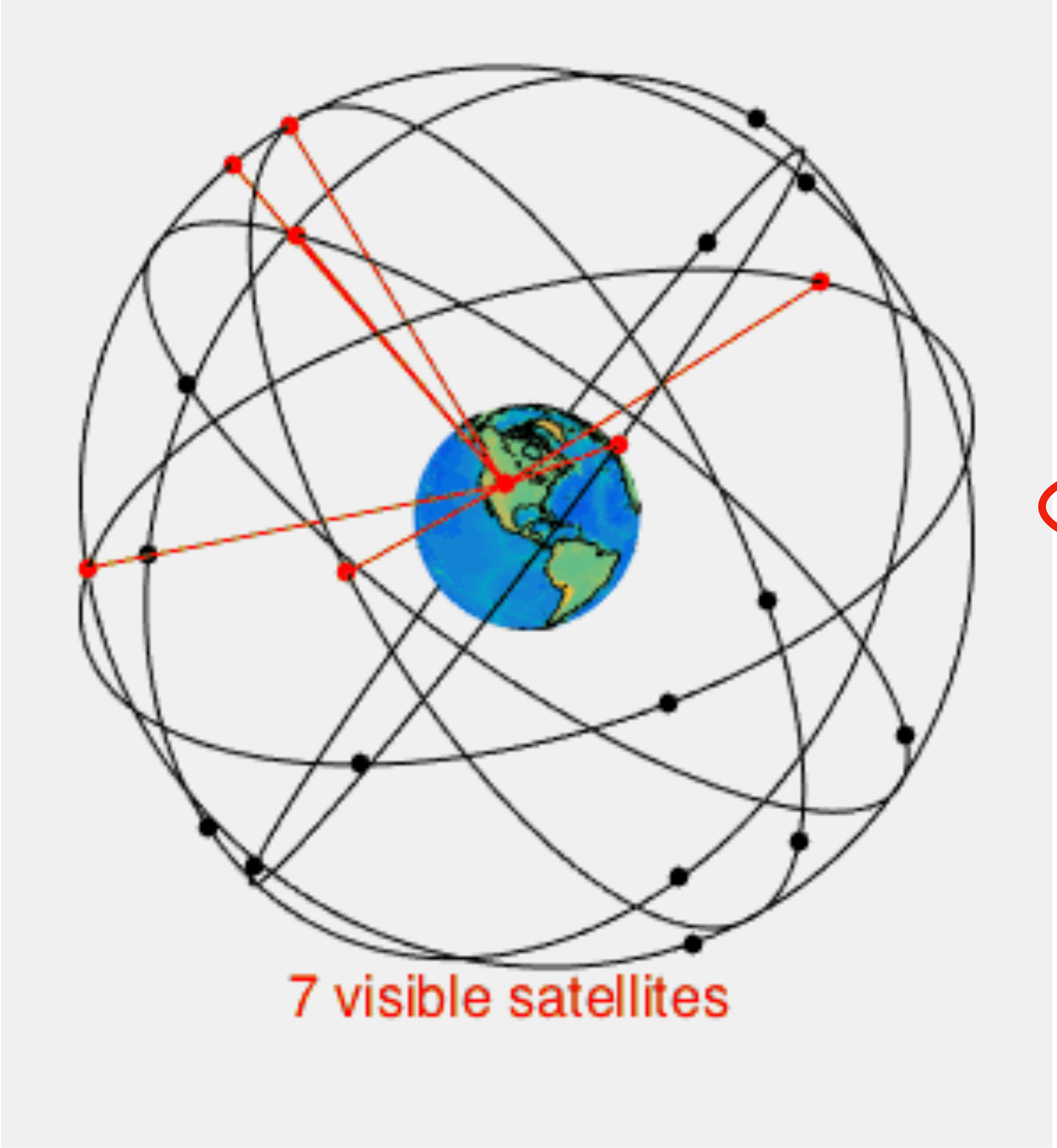

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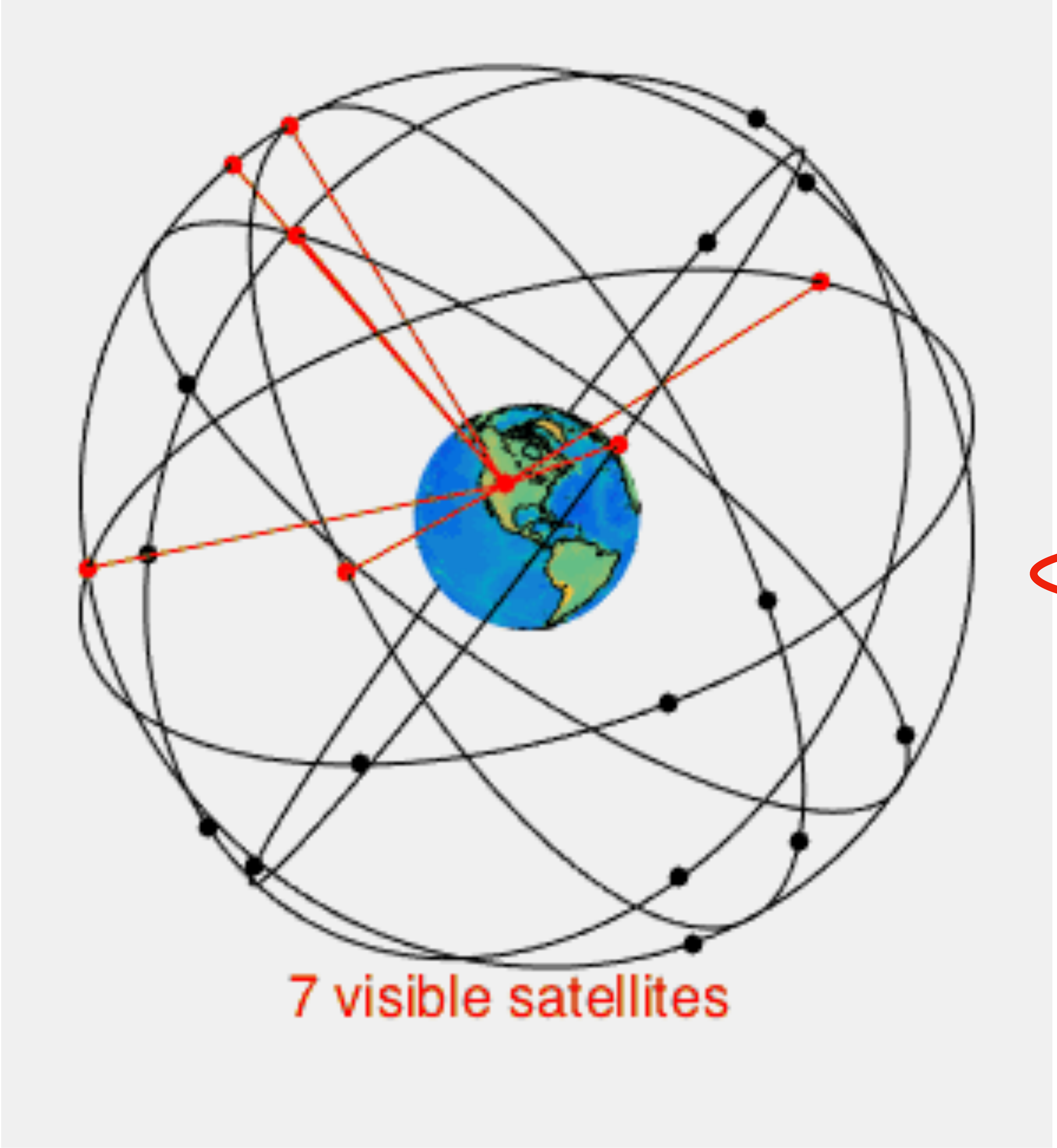

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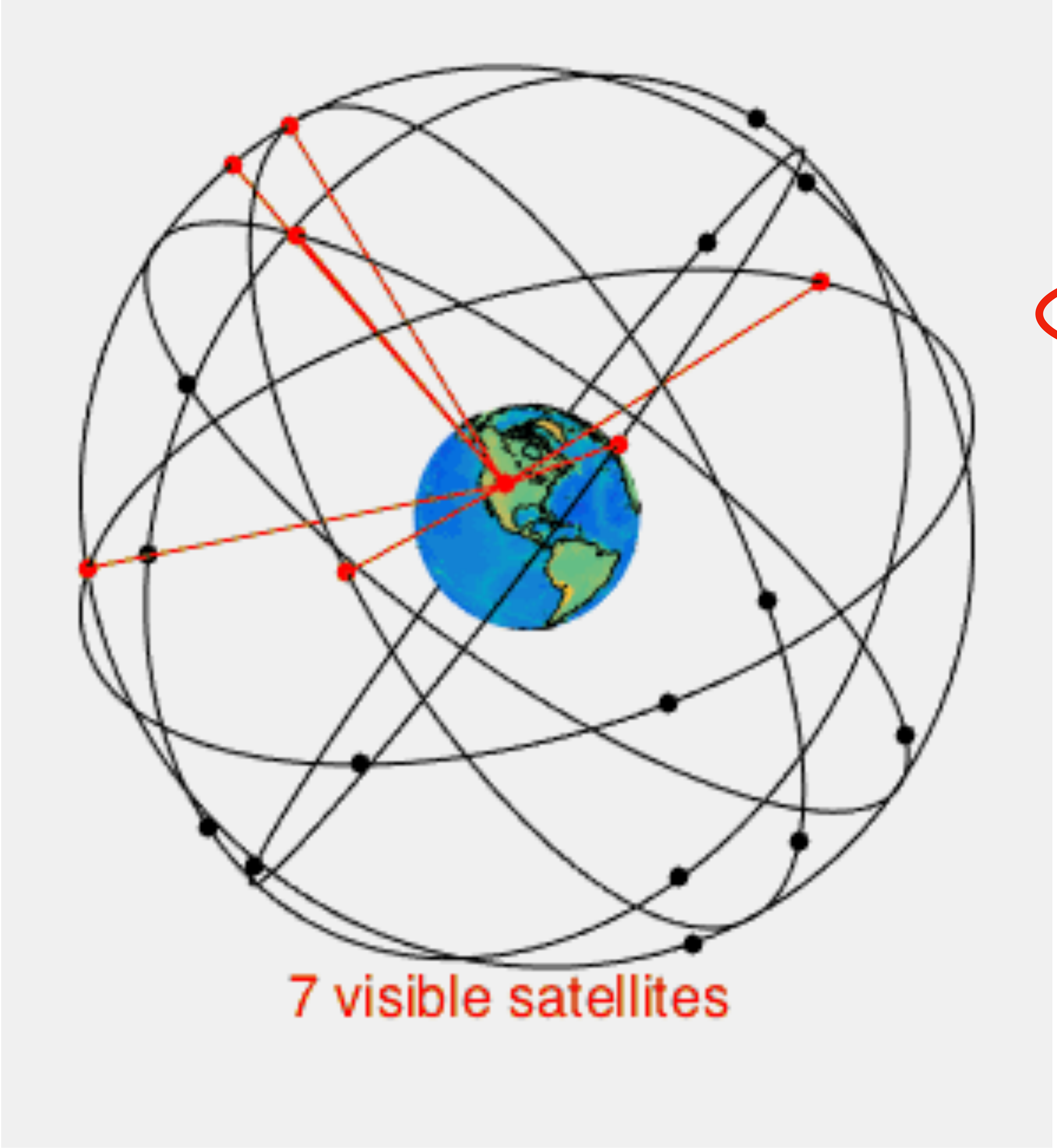

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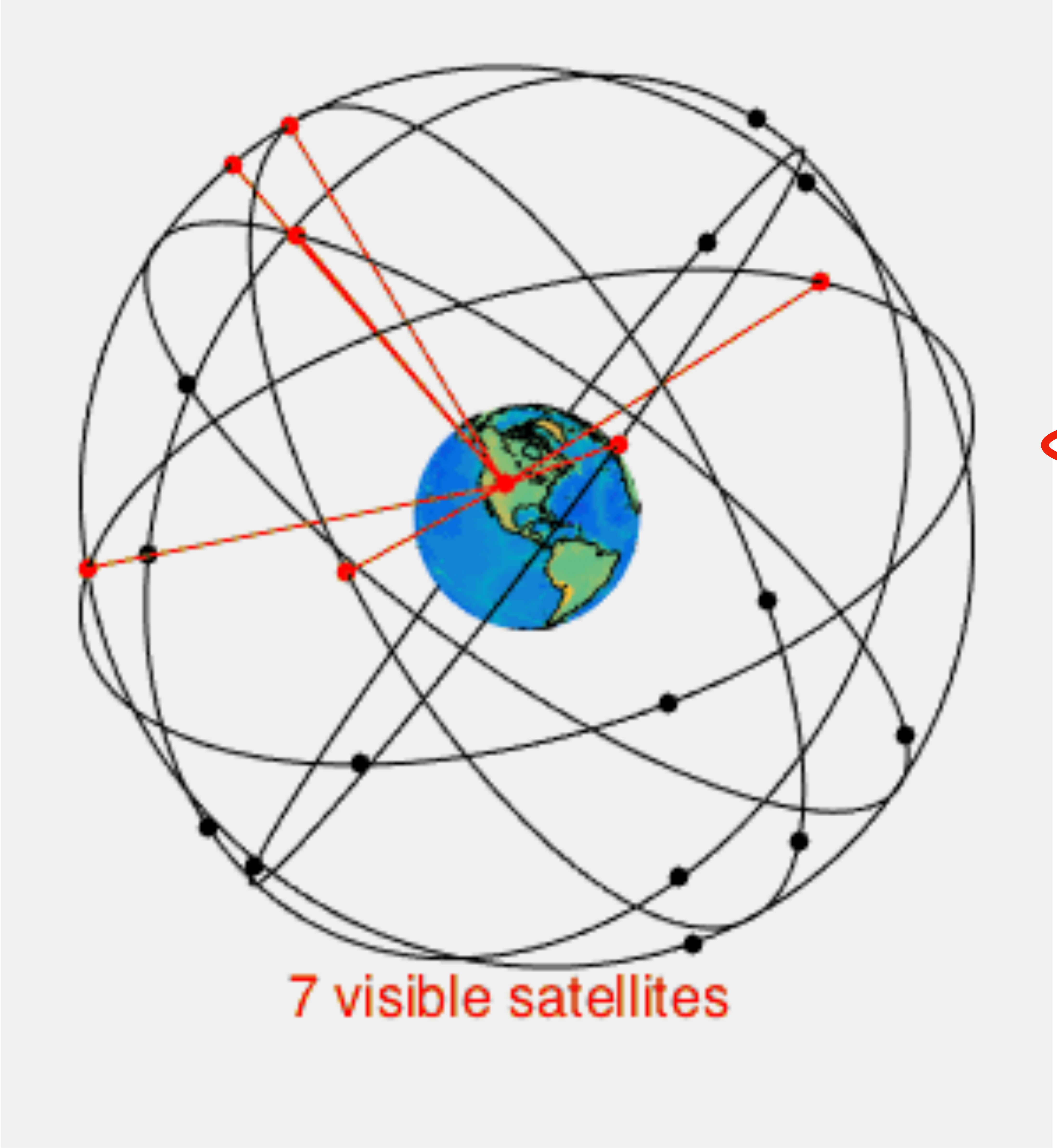

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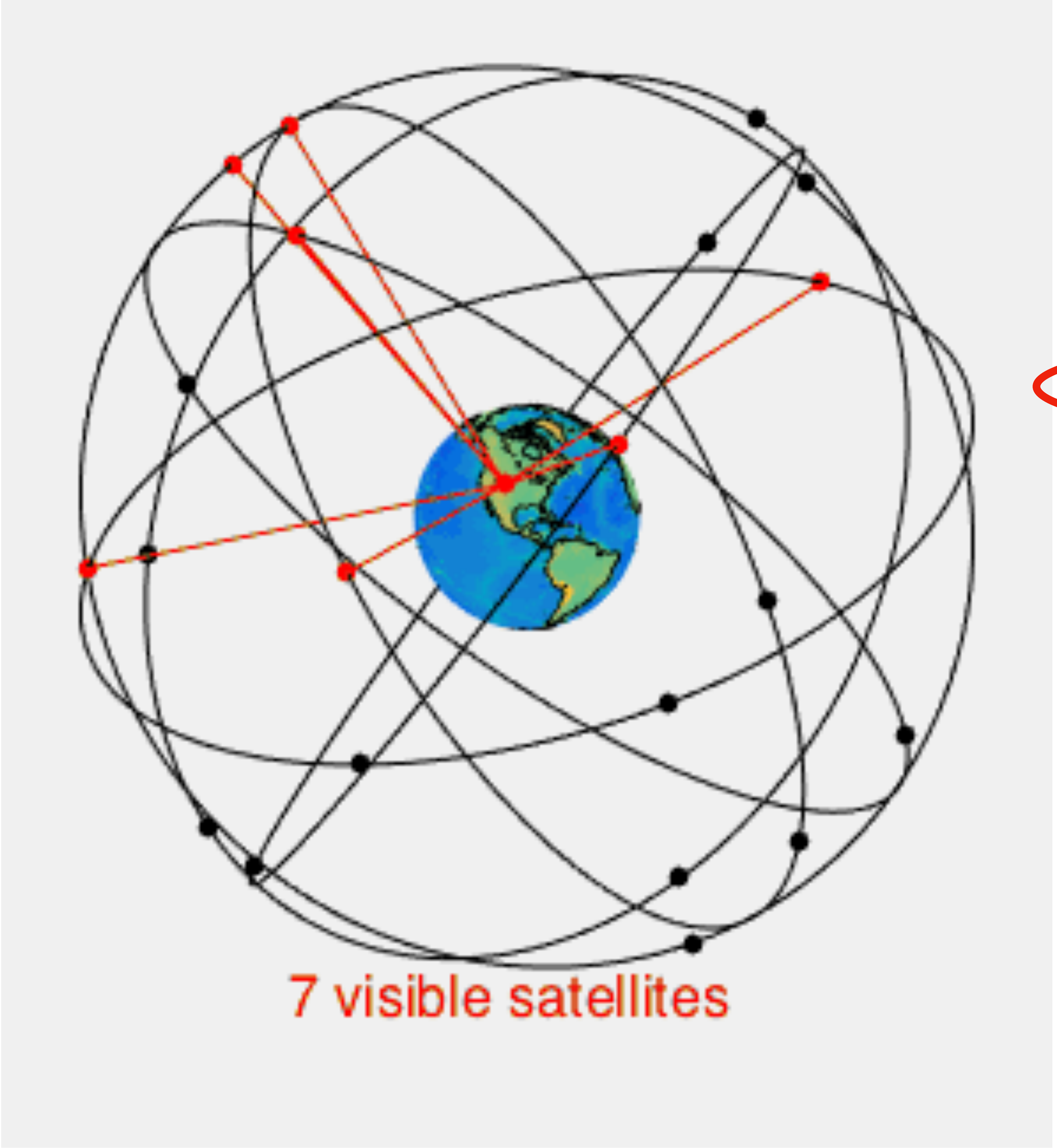

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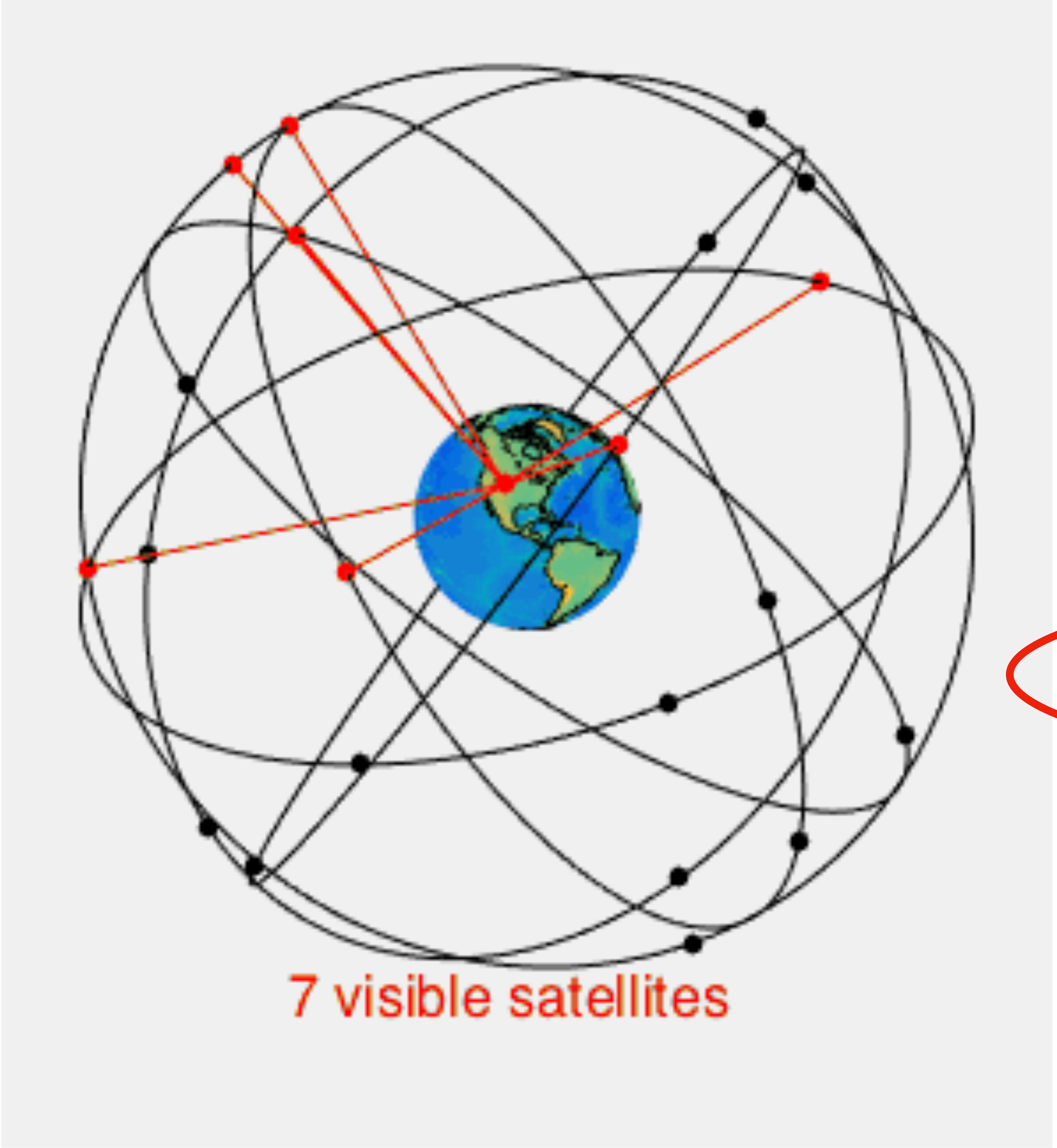

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Cool idea: Let’s provide voice and data coverage to cell phones and pagers everywhere on the surface of the Earth.

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More specifically: What do you think?

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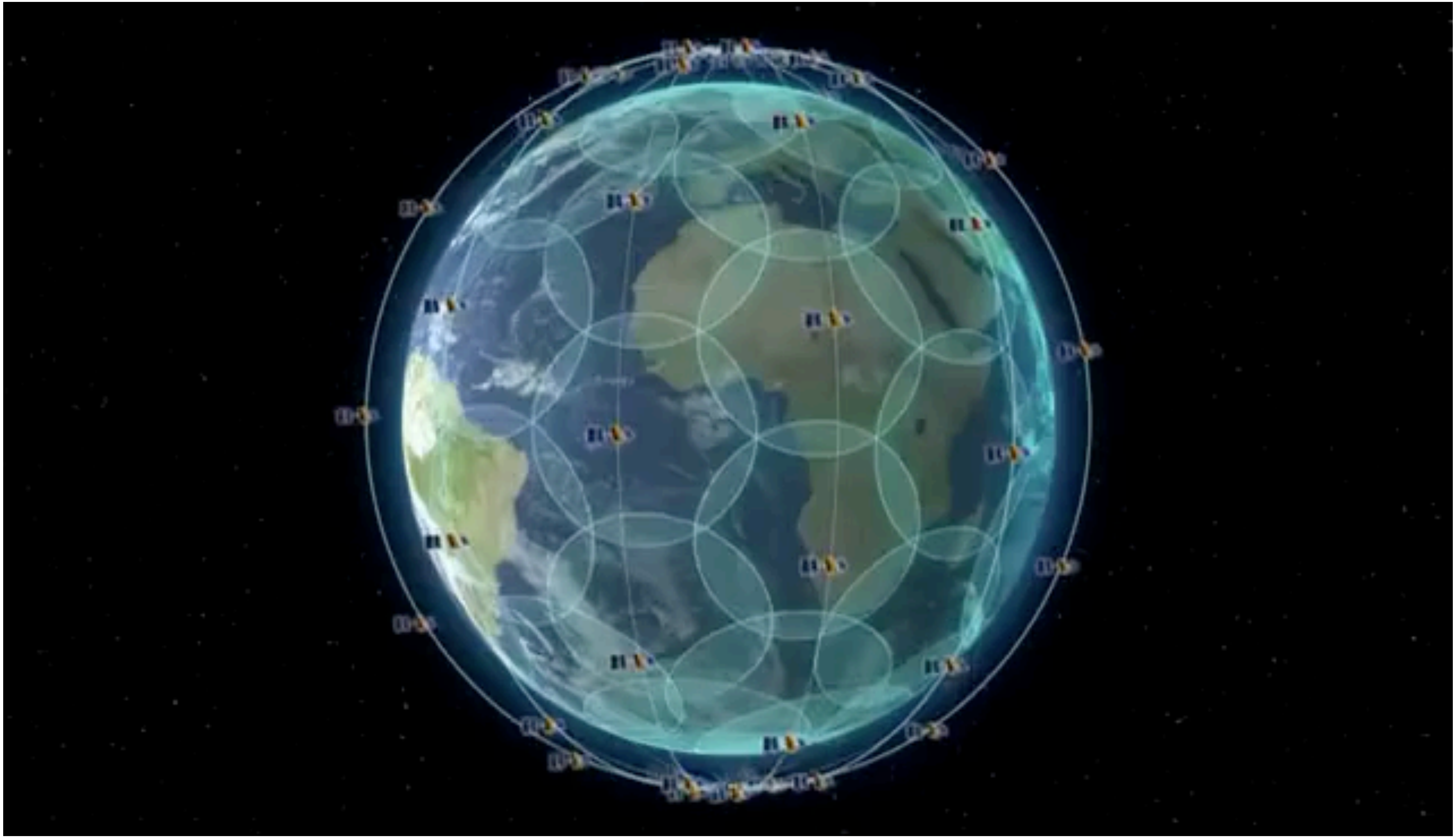
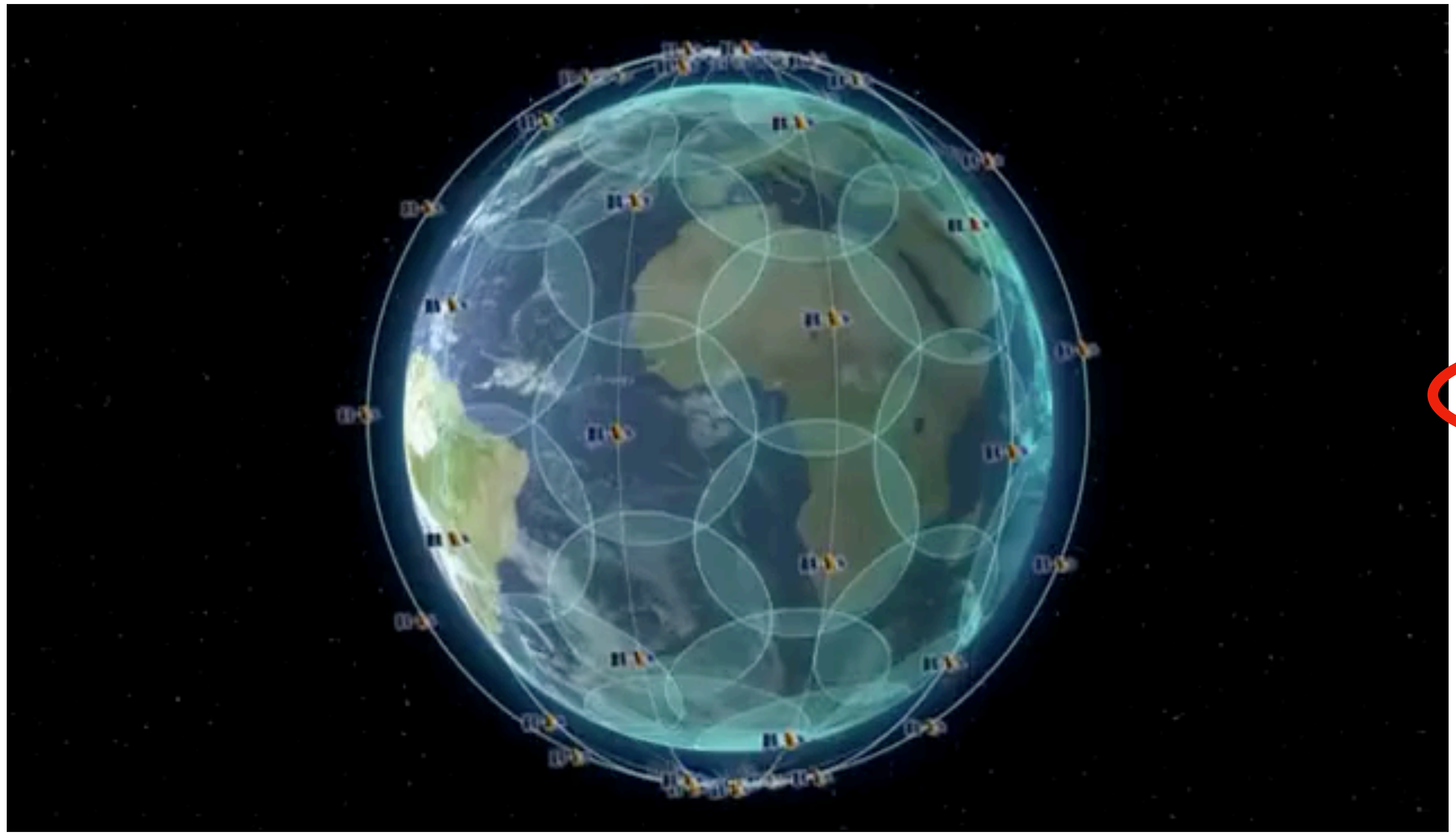
Answer: Cheating again, here’s the constellation . . .



Iridium Constellation

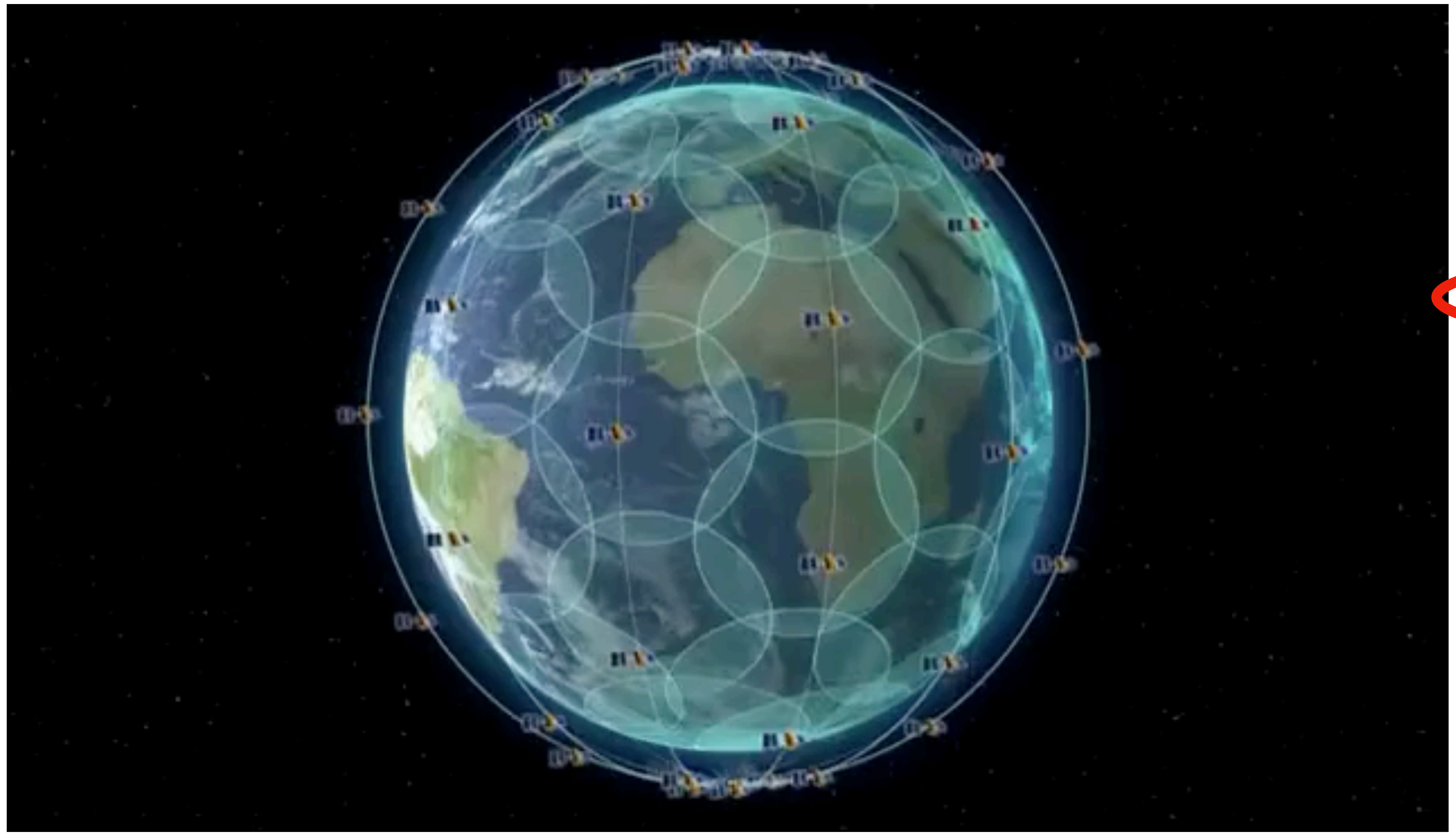


Properties:

- 6 orbital planes
- Separation of 60 degrees in the RAAN for each plane
- 86.4 degrees of inclination for each plane
- Circular orbits
- Equal distribution of 11 satellites on each plane (66 spacecraft total, plus spares)

Let's consider how each of these properties helps us meet and maintain requirements.

Iridium Constellation



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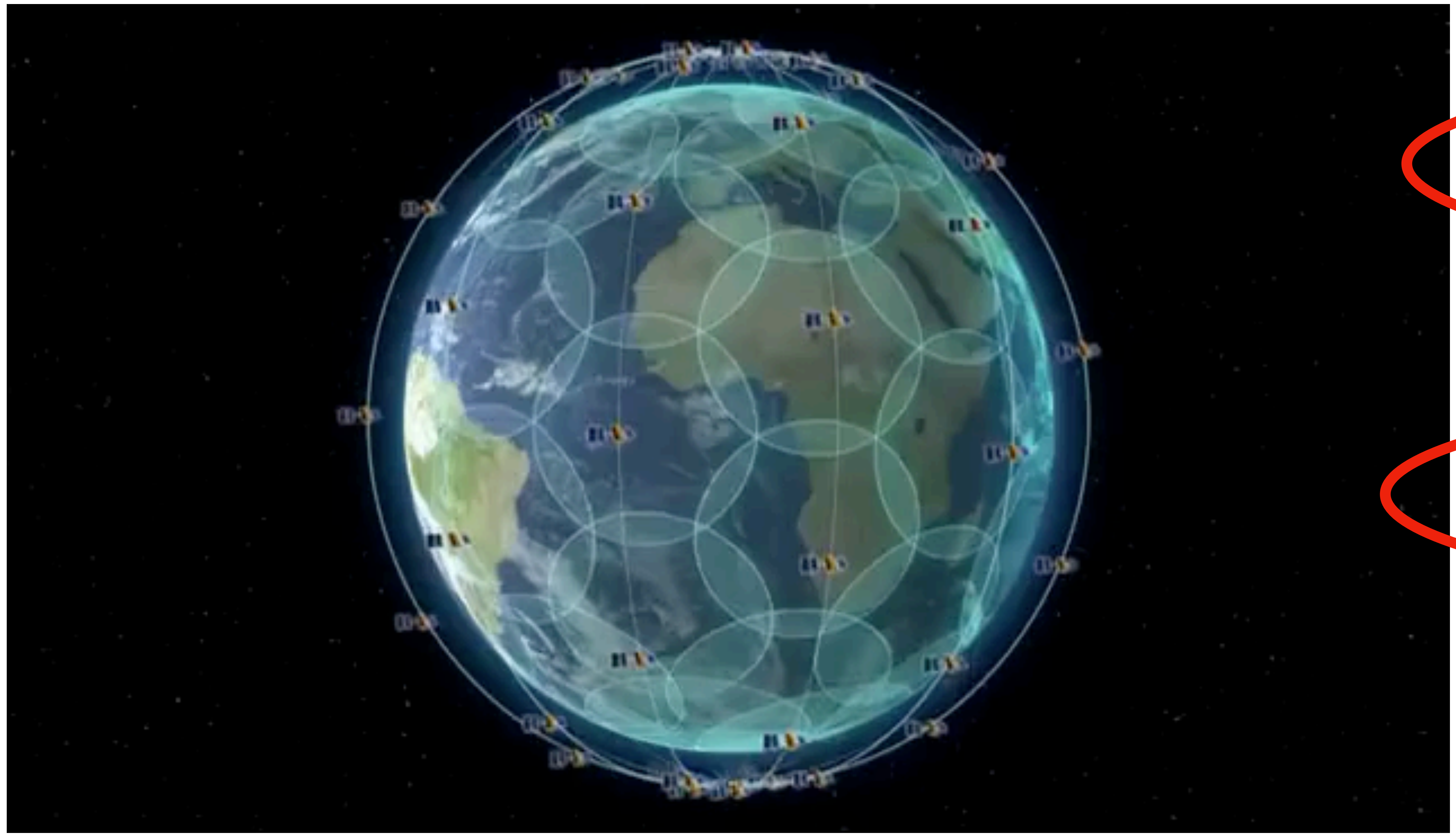

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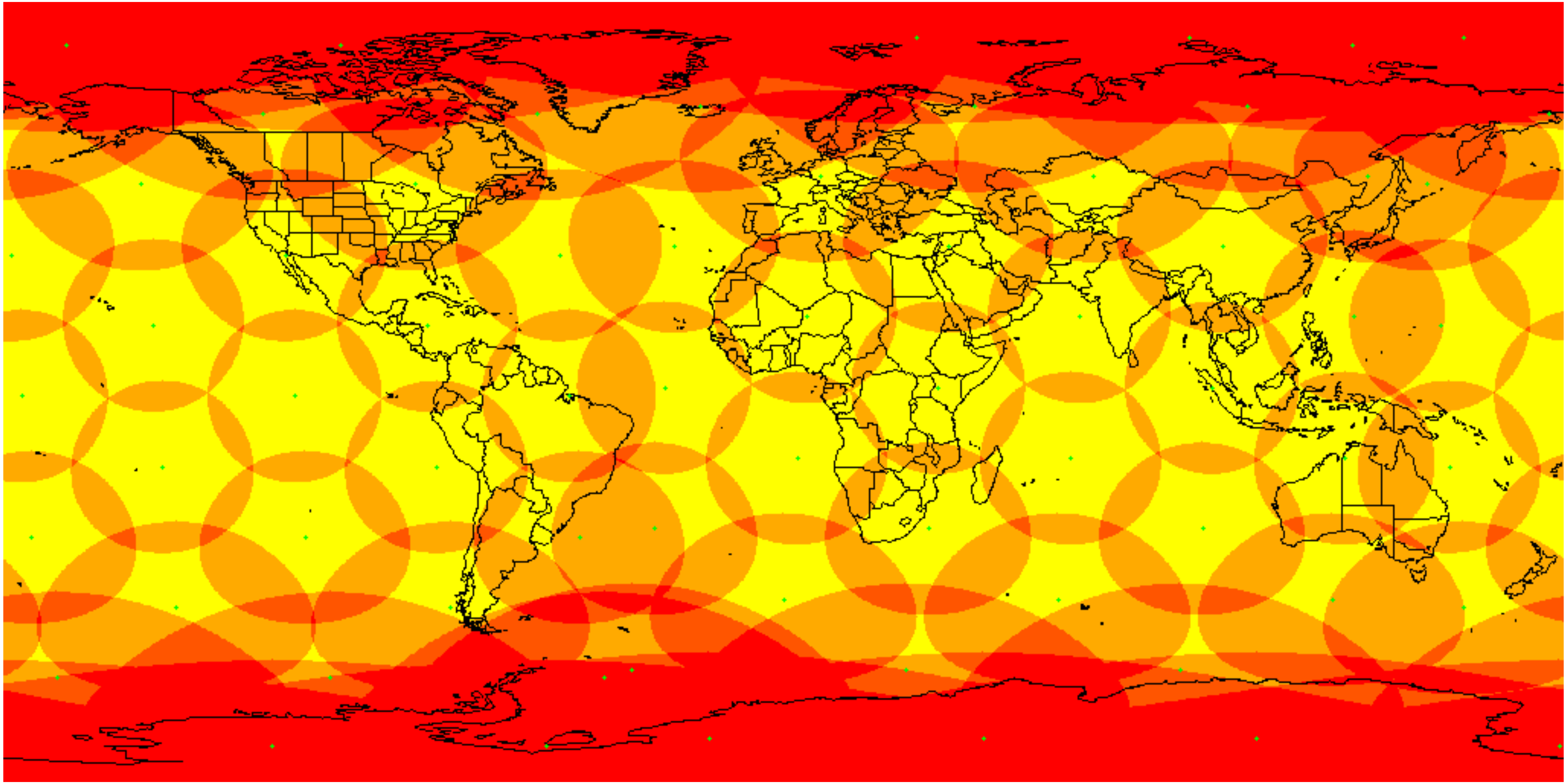
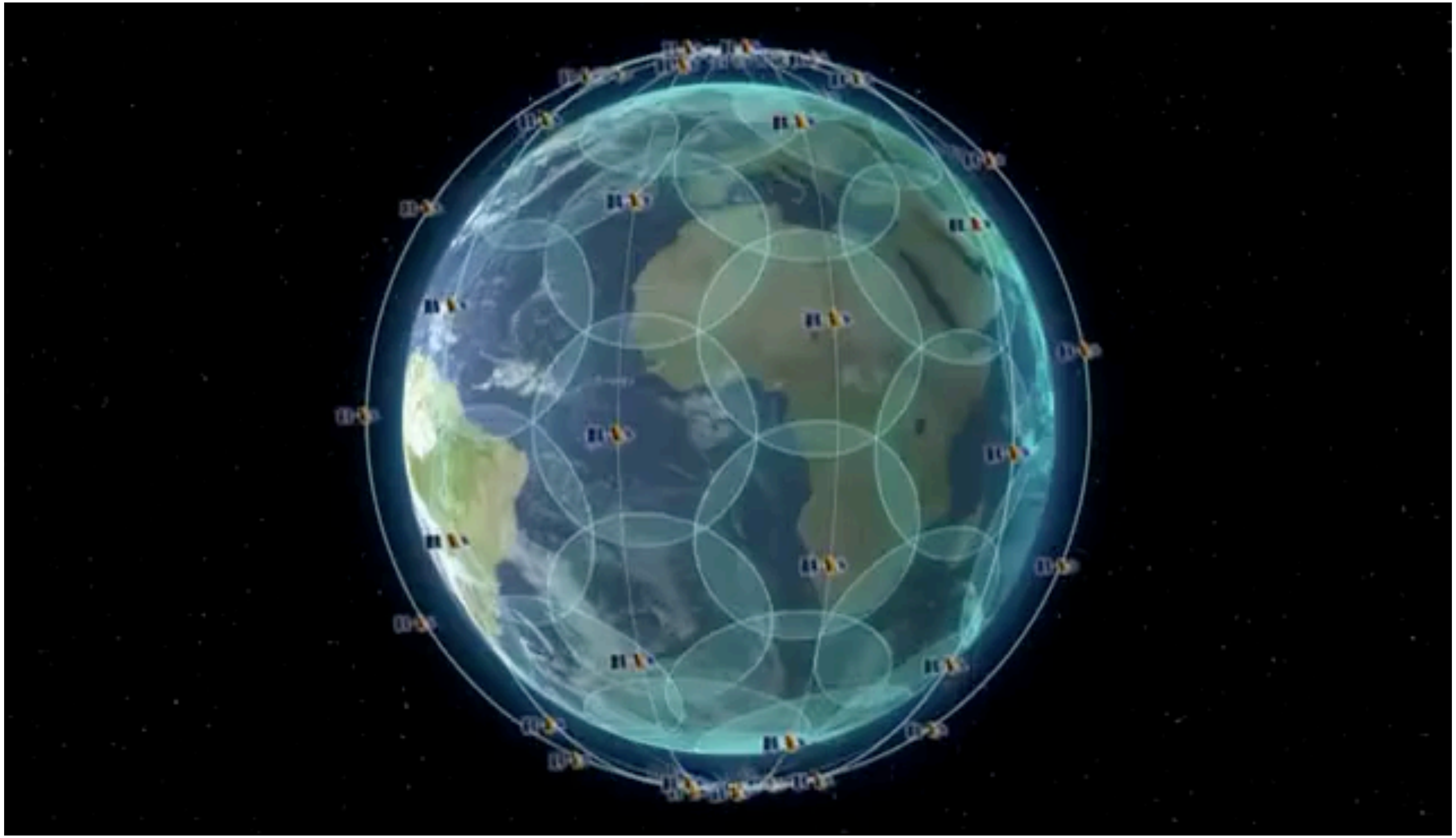


Can you spot any problems with this constellation for routing?



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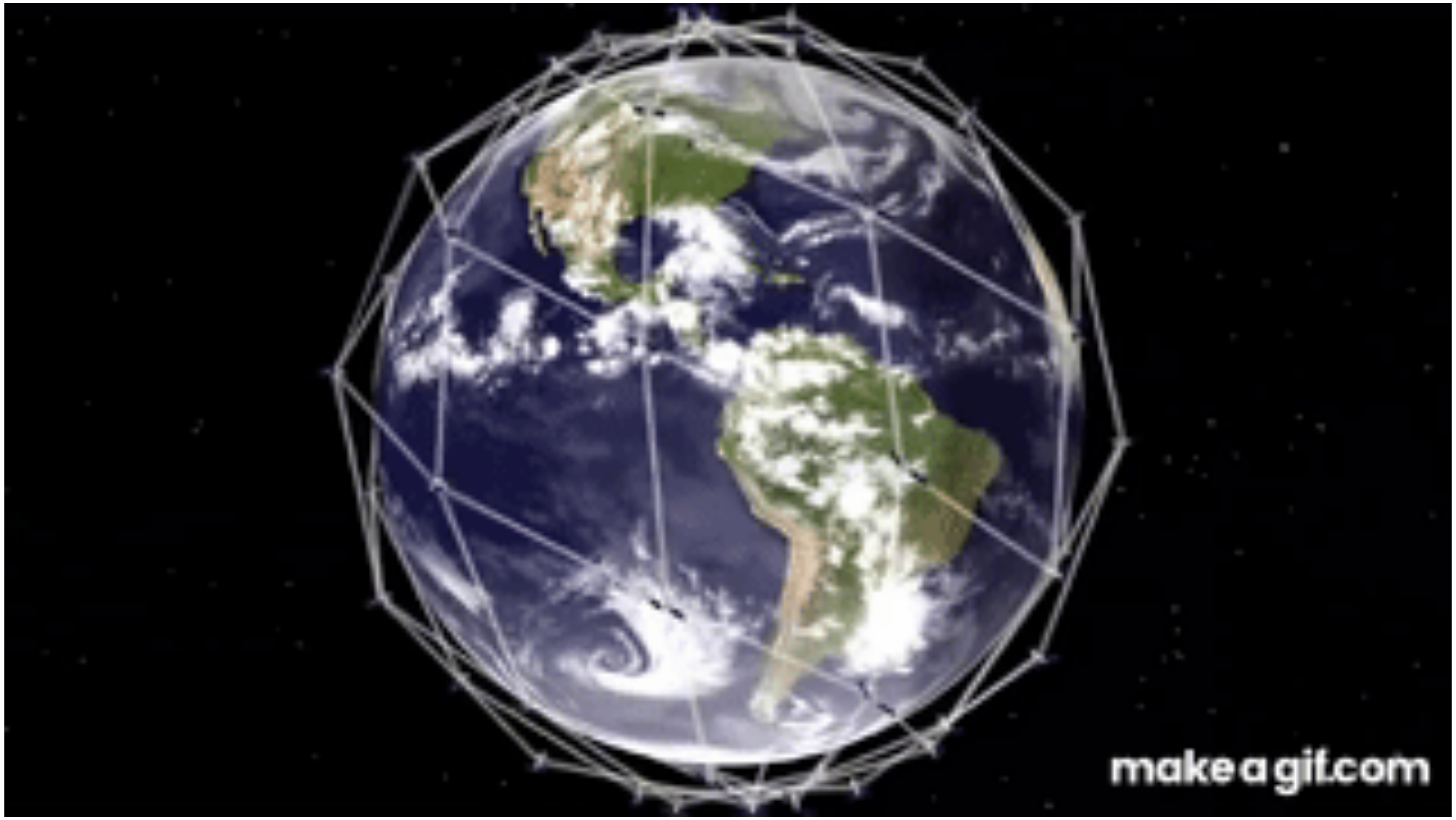

Cool idea: Let’s provide internet coverage to the entire globe.

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More specifically: Provide [bandwidth] internet connection to [lat/lon range] at [latency].

(what altitude do you expect)

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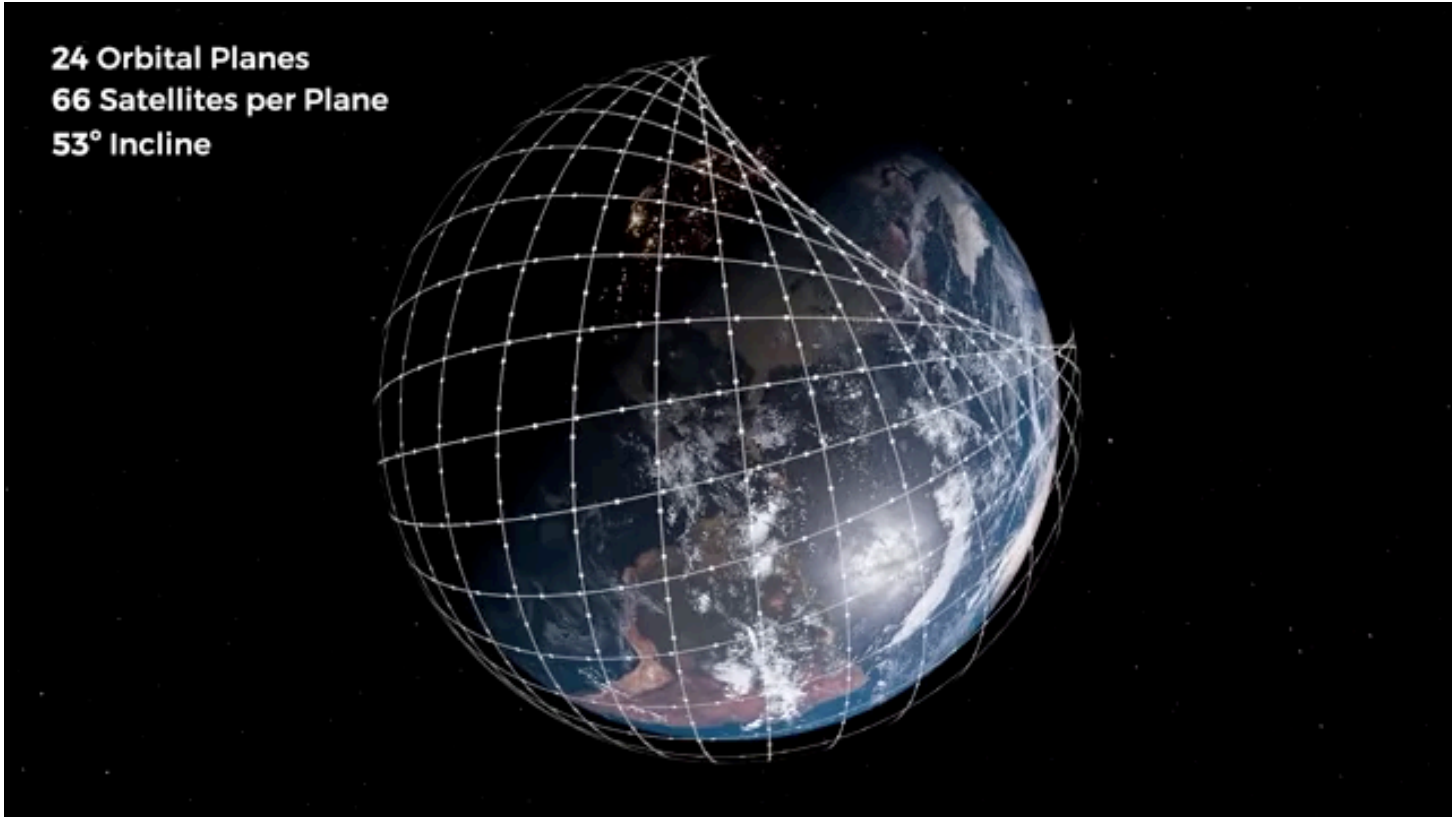
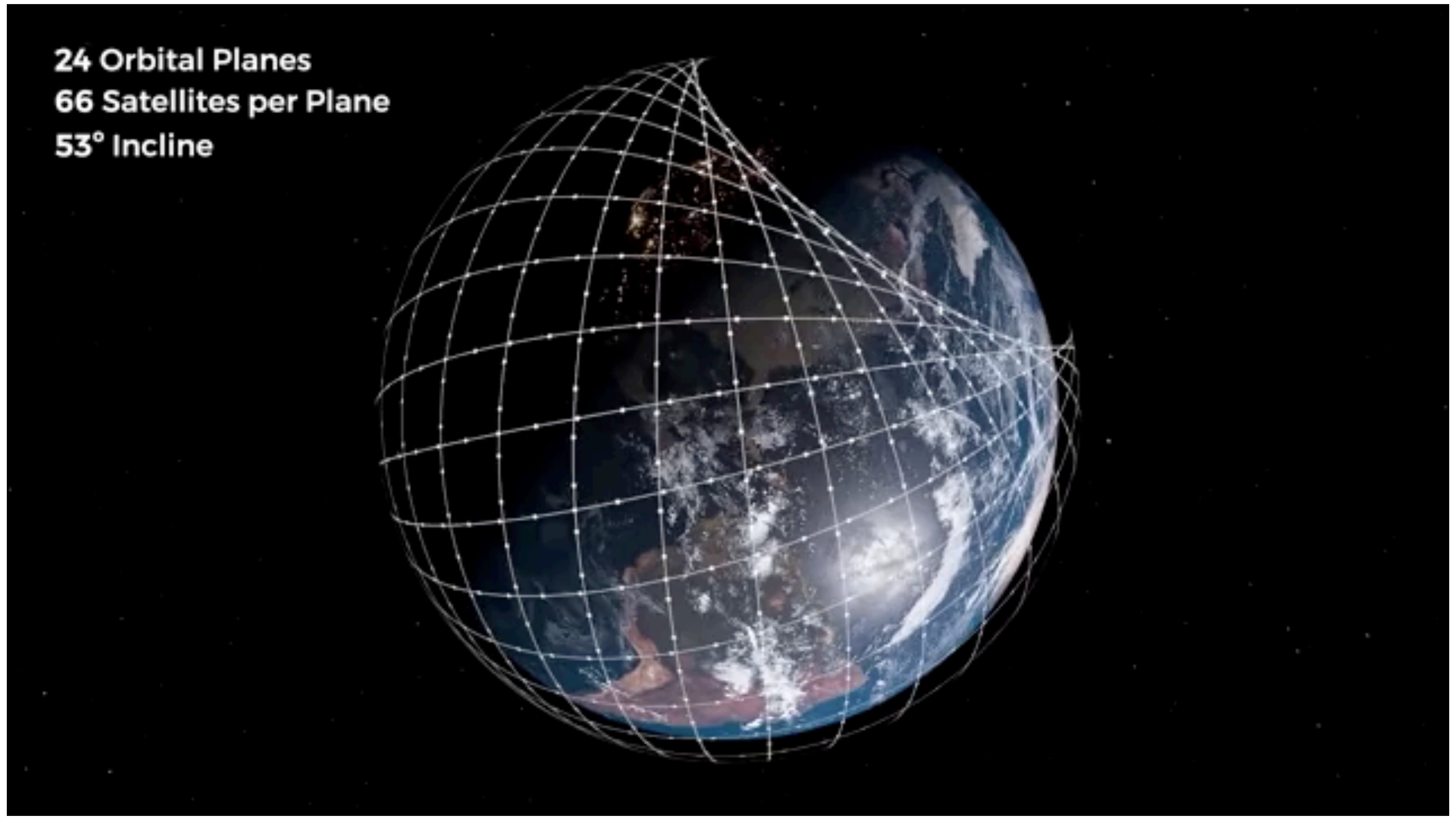
There's not that much information out there on Starlink. But, it’s too cool not to cover in a talk about constellations.

Starlink (Phase 1)



Starlink

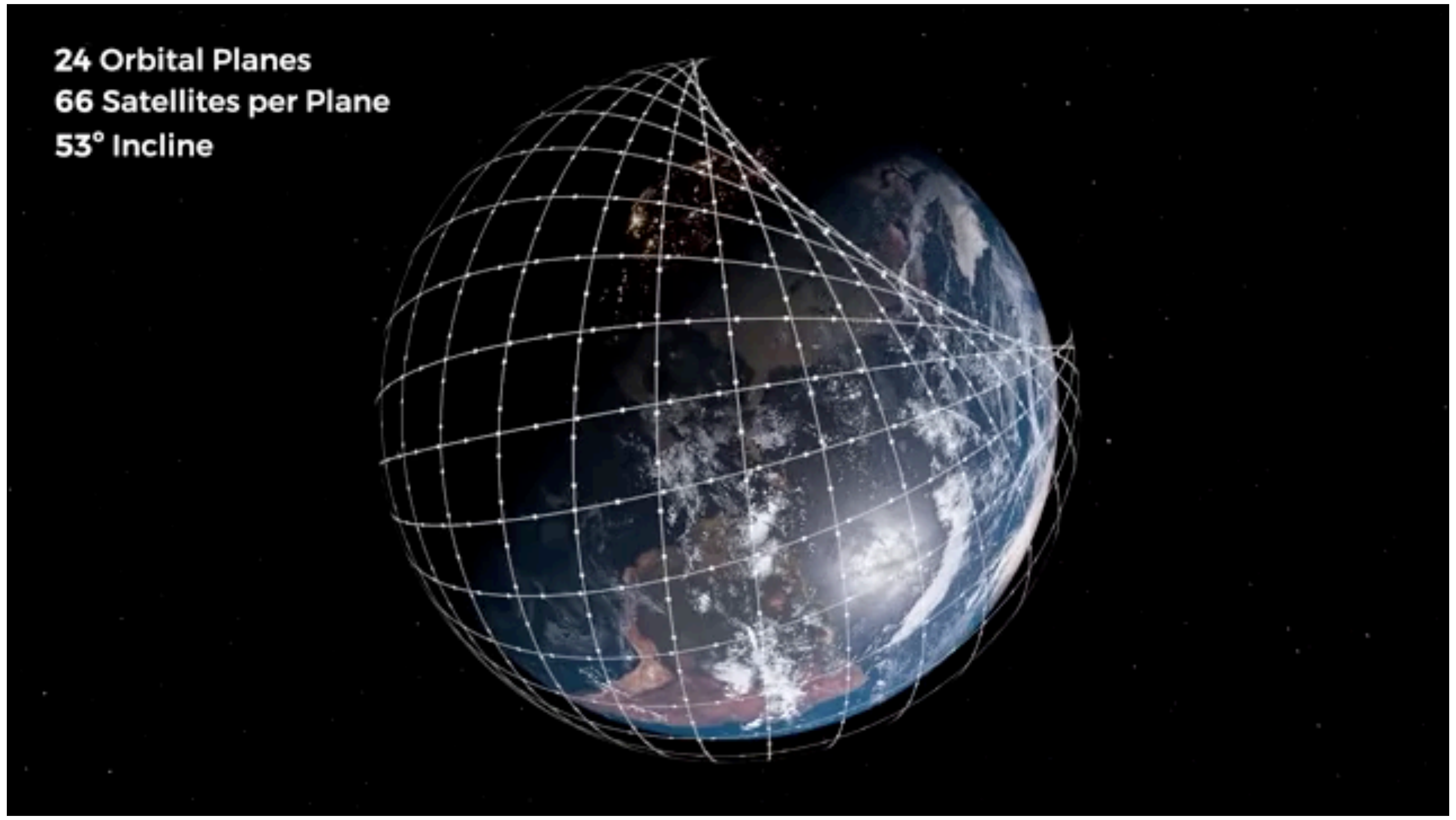


Properties (Phase 1):

- **24 orbital planes**
- **Separation of 15 degrees in the RAAN for each plane**
- **53 degrees of inclination for each plane**
- **Circular orbits**
- **Equal distribution of 66 satellites on each plane (1564 spacecraft total, plus spares)**
- **1150 km altitude**

Very possible that these numbers are out of date. If you know better, tell me.

Starlink



Some general information

- **Prioritizes east-west data transmission (future phases will improve north-south latency)**
- **Phase 1 routing connections similar to Iridium's, but with lasers**
- **Phase 1 does not provide coverage to very north/ south latitudes.**