# Propulsion and GNC

MAE 4160, 4161, 5160

V. Hunter Adams, PhD

# Today’s topics:

- Vis-viva equation
- Escape velocity
- Hohmann transfers
- Interplanetary Hohmann transfers
- Flybys
- Rocket equation

# Vis-viva equation

$$\frac{v^2}{2} - \frac{GM}{r} = -\frac{GM}{2a}$$

# Vis-viva equation

velocity
Gravitational Constant
mass of planet

$$\frac{v^2}{2} - \frac{GM}{r} = -\frac{GM}{2a}$$

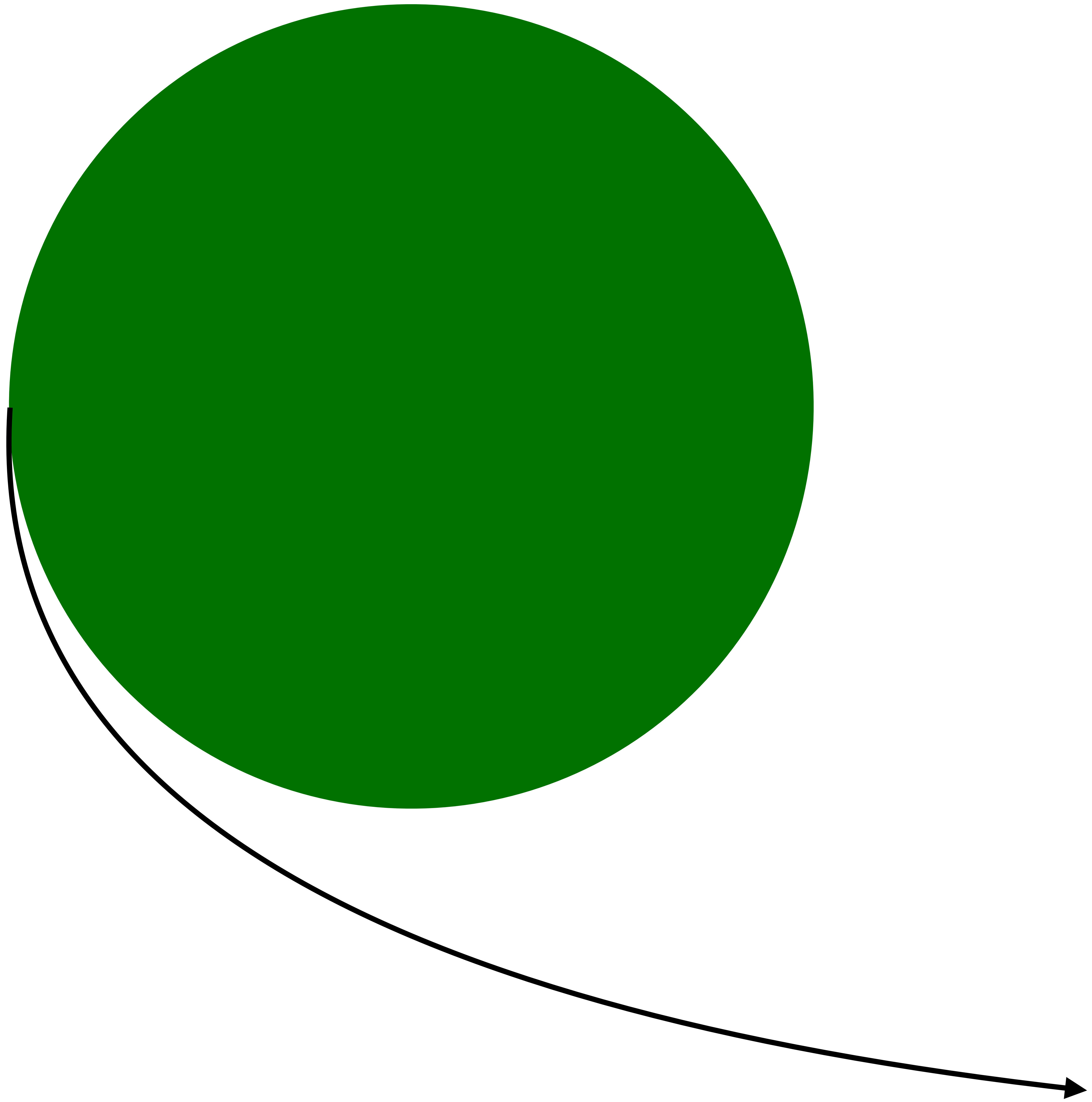
distance from planet
semimajor axis

**Comes from conservation of energy and angular momentum.**

The vis-viva equation is useful for calculating:

- Escape velocity
- Hohmann transfers
- Interplanetary Hohmann transfers

# Escape velocity

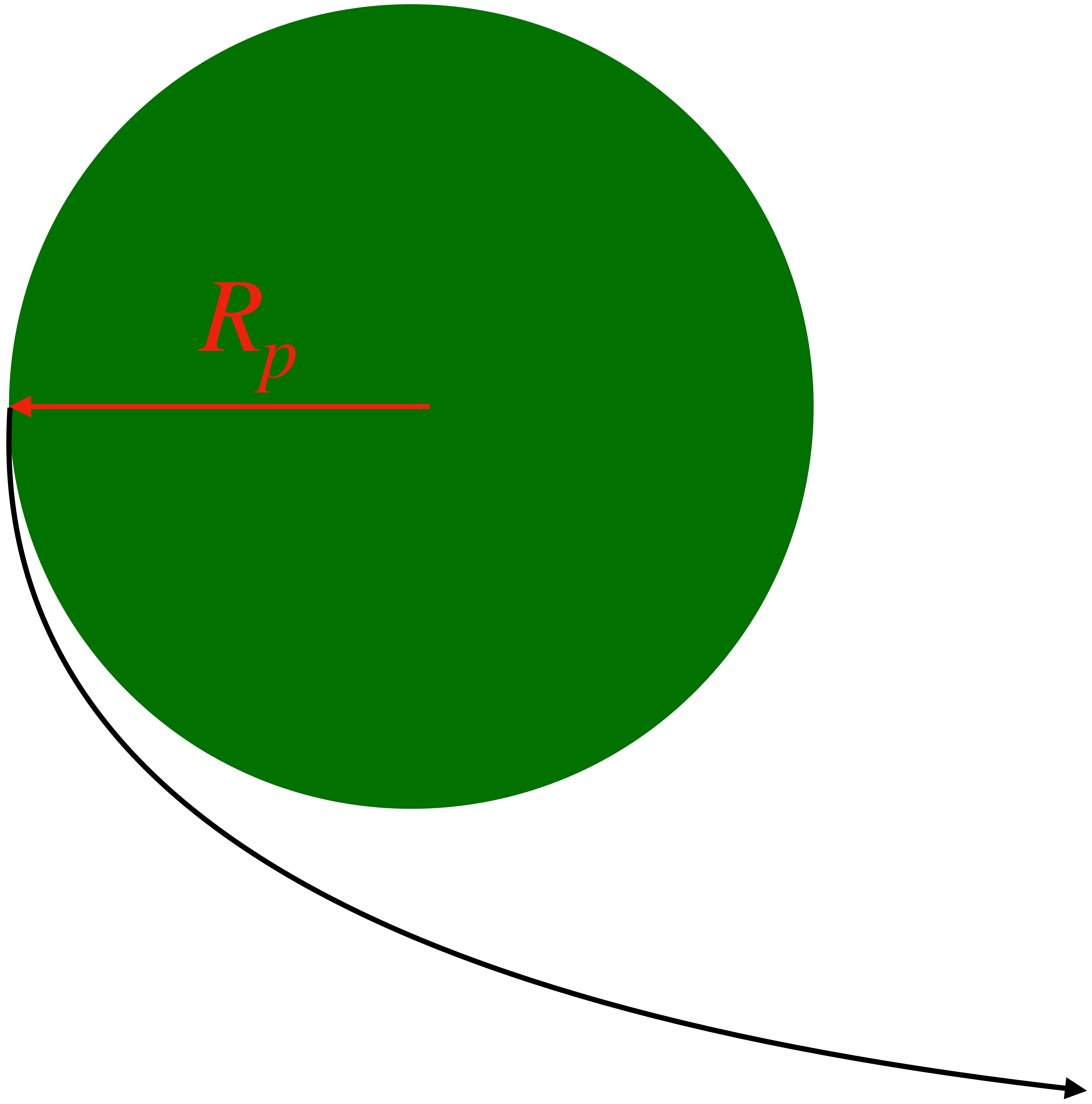


1. Solve vis-viva for velocity

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

2. What is *r* ? What is *a* ? Hint: which conic section is this trajectory?

# Escape velocity

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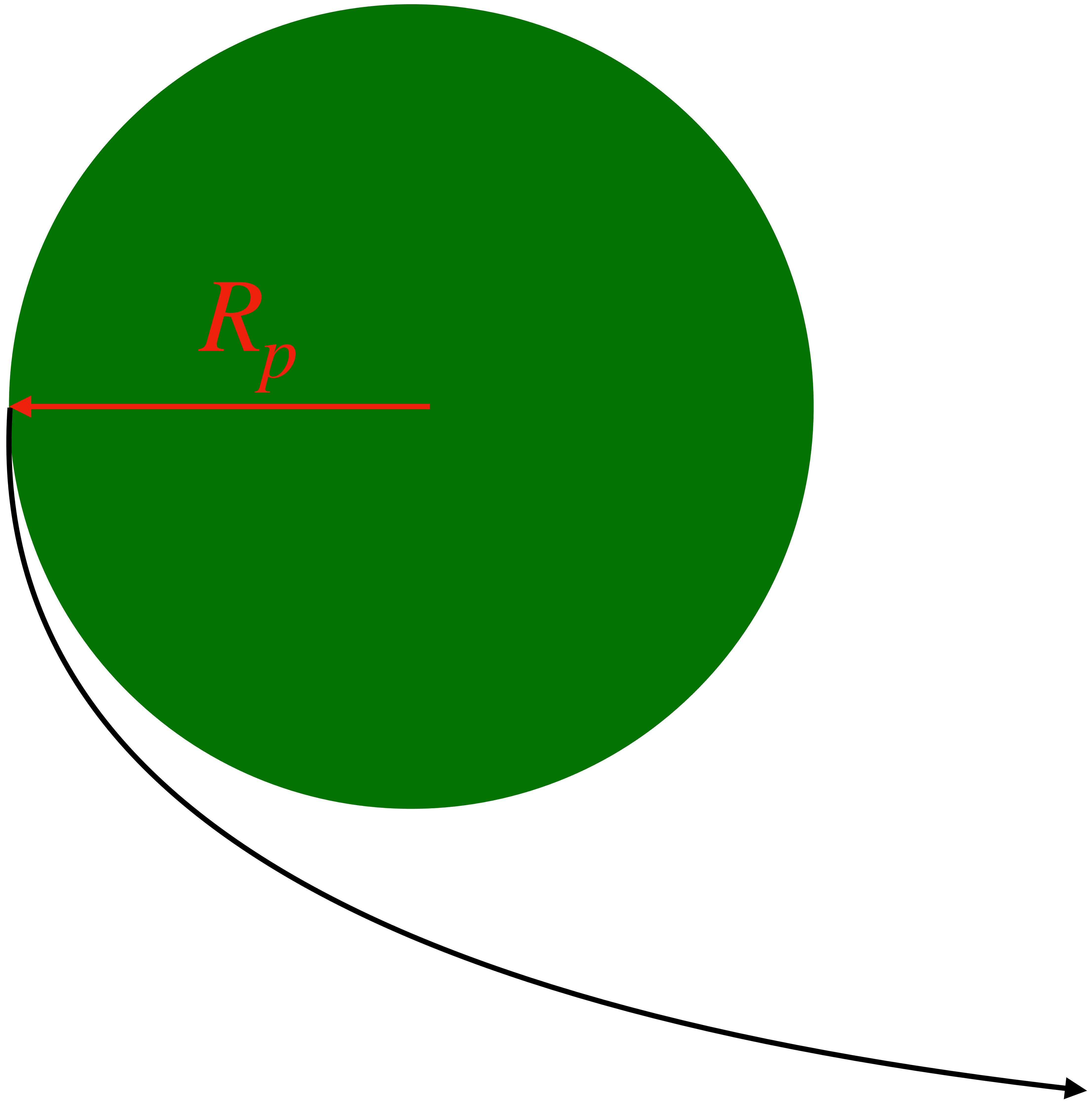
$$r = R_p$$
$$a = \infty$$

# Escape velocity




1. Solve vis-viva for velocity

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

2. What is *r* ? What is *a* ? Hint: which conic section is this trajectory?

$$r = R_p$$
$$a = \infty$$

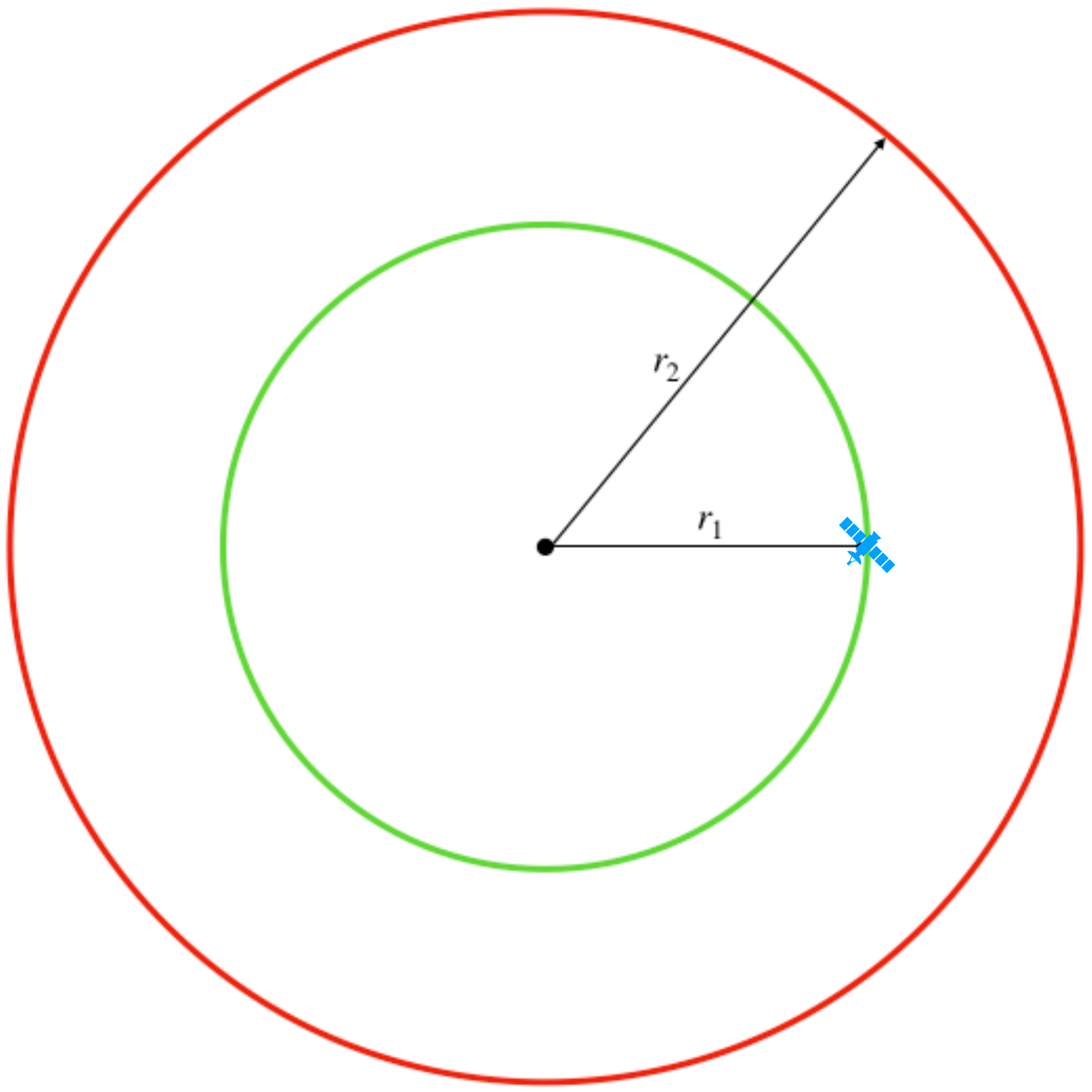
3. Substitute and solve.

$$v^2 = GM\left(\frac{2}{R_p} - \frac{1}{\infty}\right) \longrightarrow v_{esc} = \sqrt{\frac{2GM}{R_p}}$$

The vis-viva equation is useful for calculating:

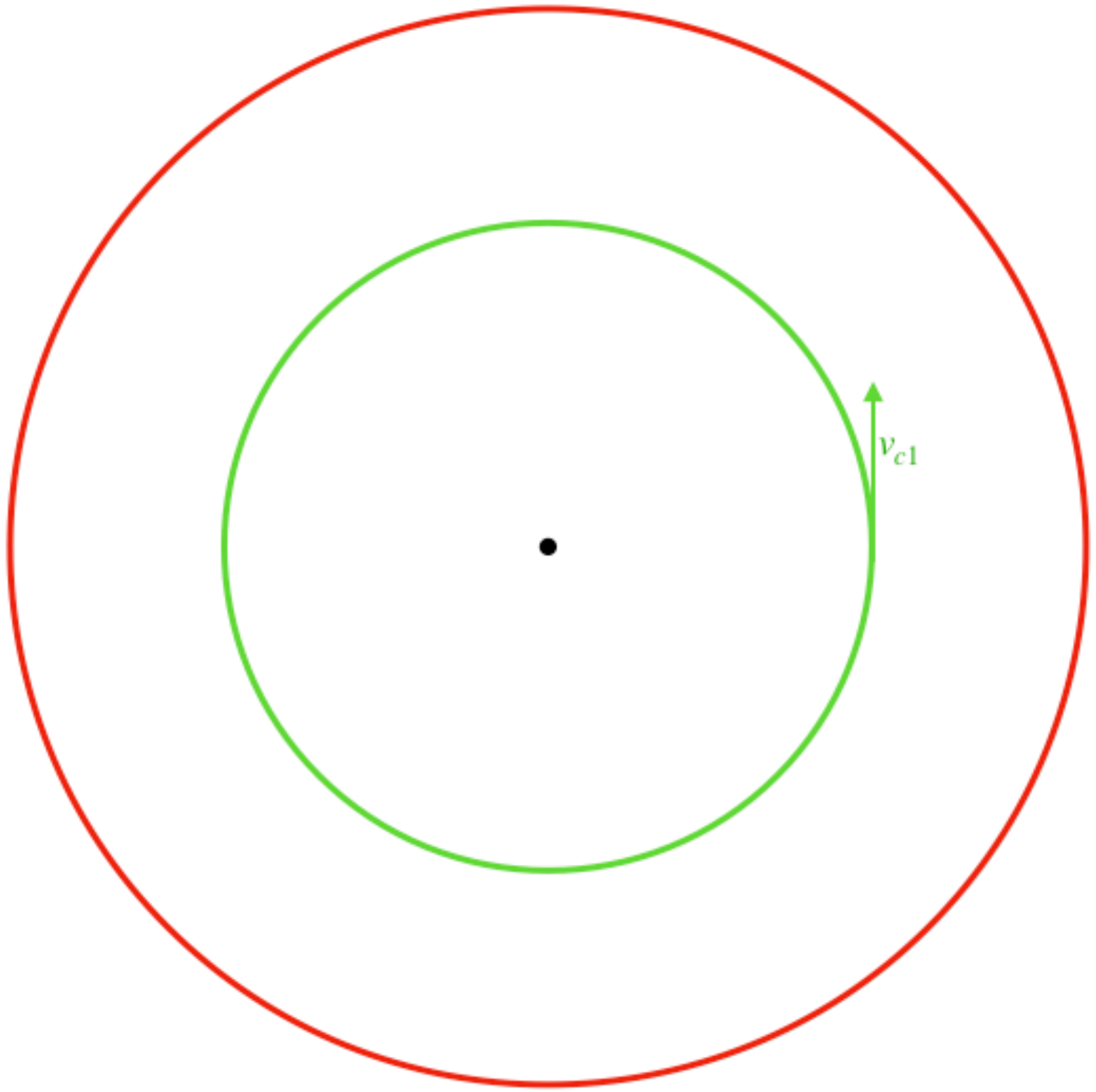
- Escape velocity
- Hohmann transfers
- Interplanetary Hohmann transfers

# Hohmann transfer




**<u>Goal</u>:** Move a spacecraft from a circular orbit of radius r1 to a circular orbit of radius r2

# Hohmann transfer



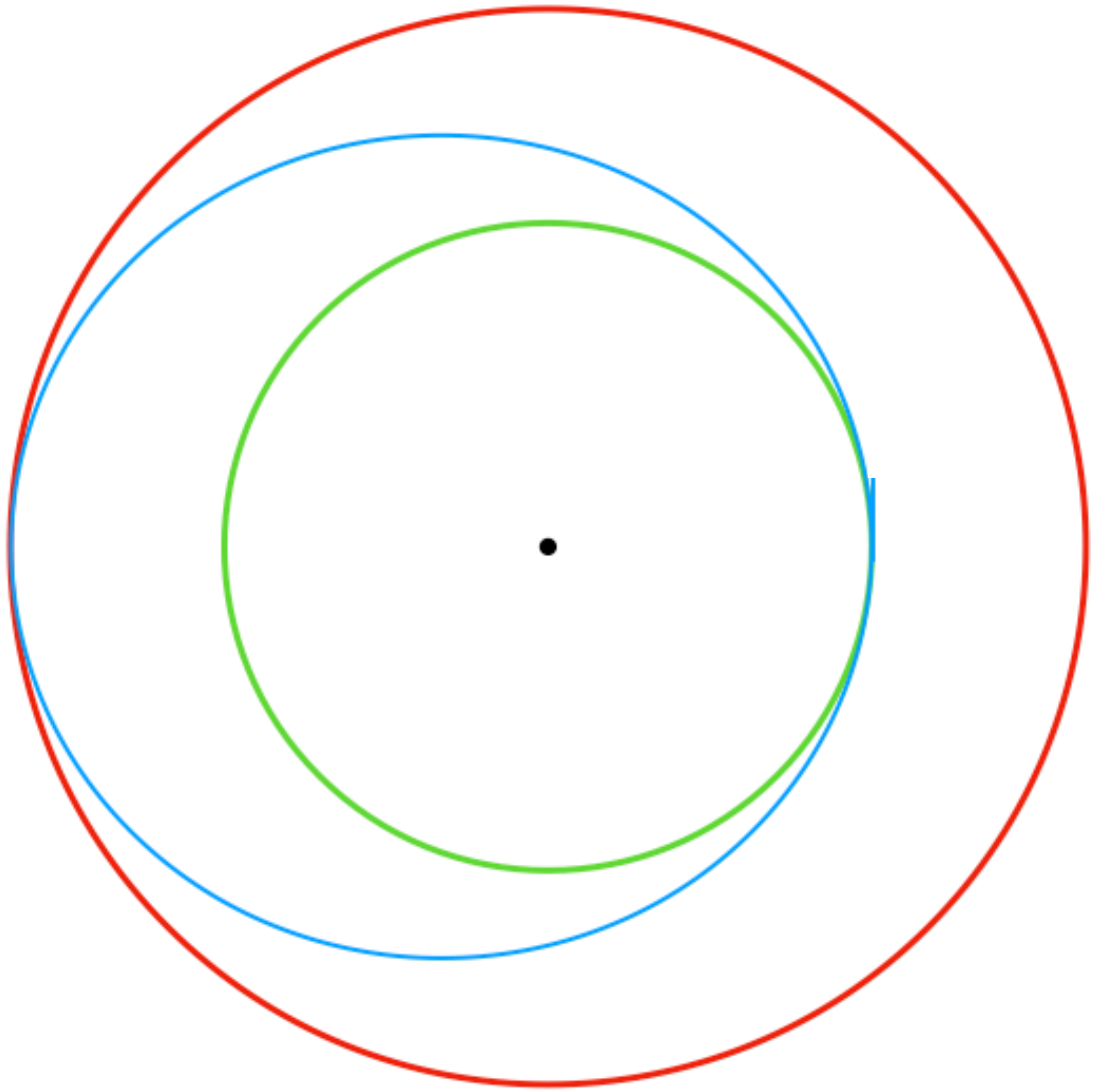

1. Calculate the velocity of the spacecraft on the initial circular orbit of radius r1 using the vis-viva equation

$$v_{c1} = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$= \sqrt{GM\left(\frac{2}{r_1} - \frac{1}{r_1}\right)}$$

$$= \sqrt{\frac{GM}{r_1}}$$

# Hohmann transfer

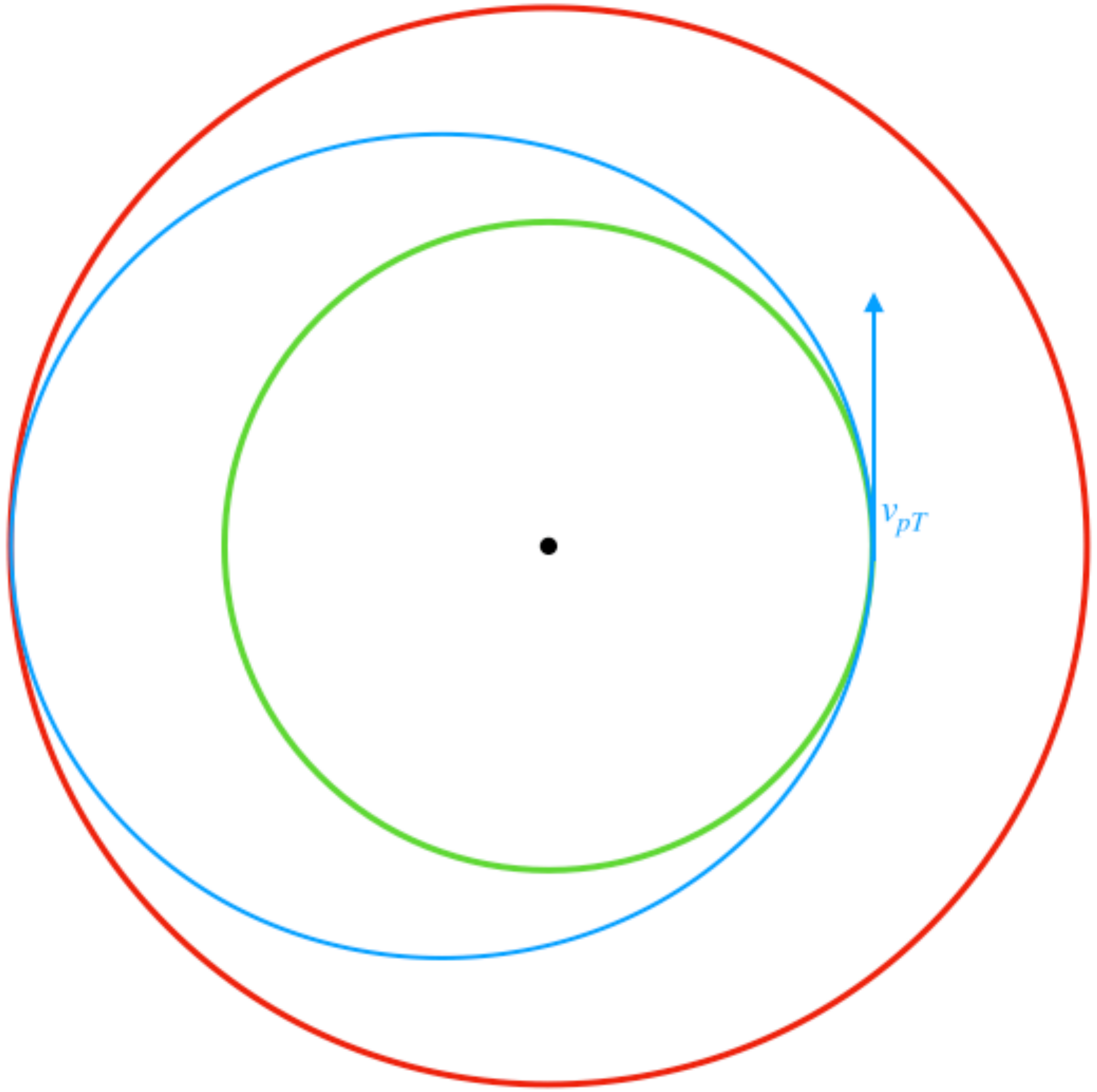


2.Calculate the elliptical transfer orbit semi major axis and eccentricity

$$a_T = \frac{r_1 + r_2}{2}$$

$$e_T = \frac{r_1 - r_2}{r_1 + r_2}$$

# Hohmann transfer




3. Calculate the perigee velocity of the transfer orbit using vis-viva

$$v_{pT} = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

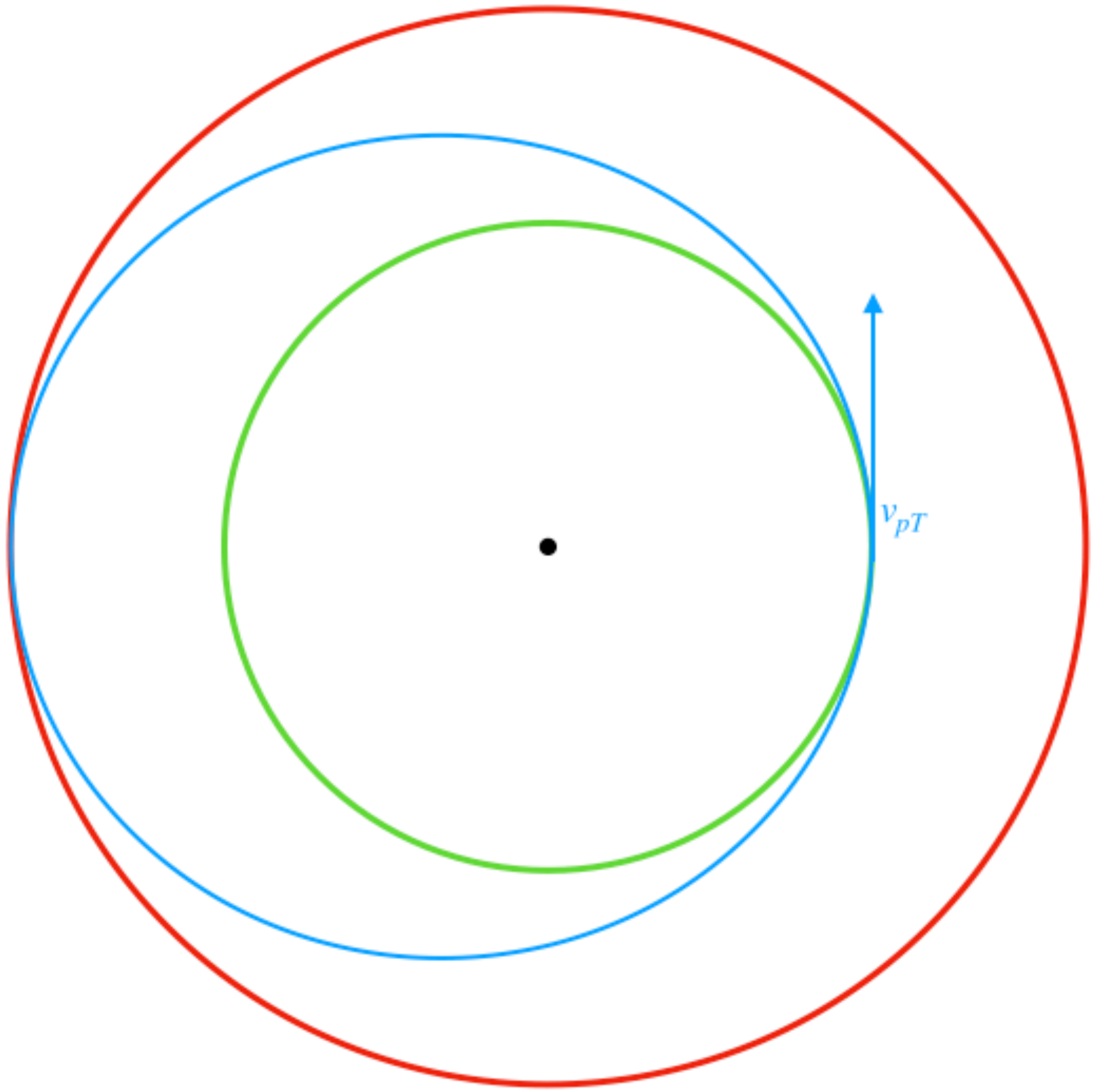
$$= \sqrt{GM\left(\frac{2}{r_1} - \frac{1}{a_T}\right)}$$

$$= \sqrt{GM\left(\frac{2}{r_1} - \frac{2}{r_1 + r_2}\right)}$$

$$= \sqrt{\frac{2GM}{r_1}\left(1 - \frac{1}{1 + \frac{r_2}{r_1}}\right)}$$

$$= \sqrt{\frac{GM}{r_1}}\sqrt{2\left(1 - \frac{1}{1 + \frac{r_2}{r_1}}\right)}$$

# Hohmann transfer



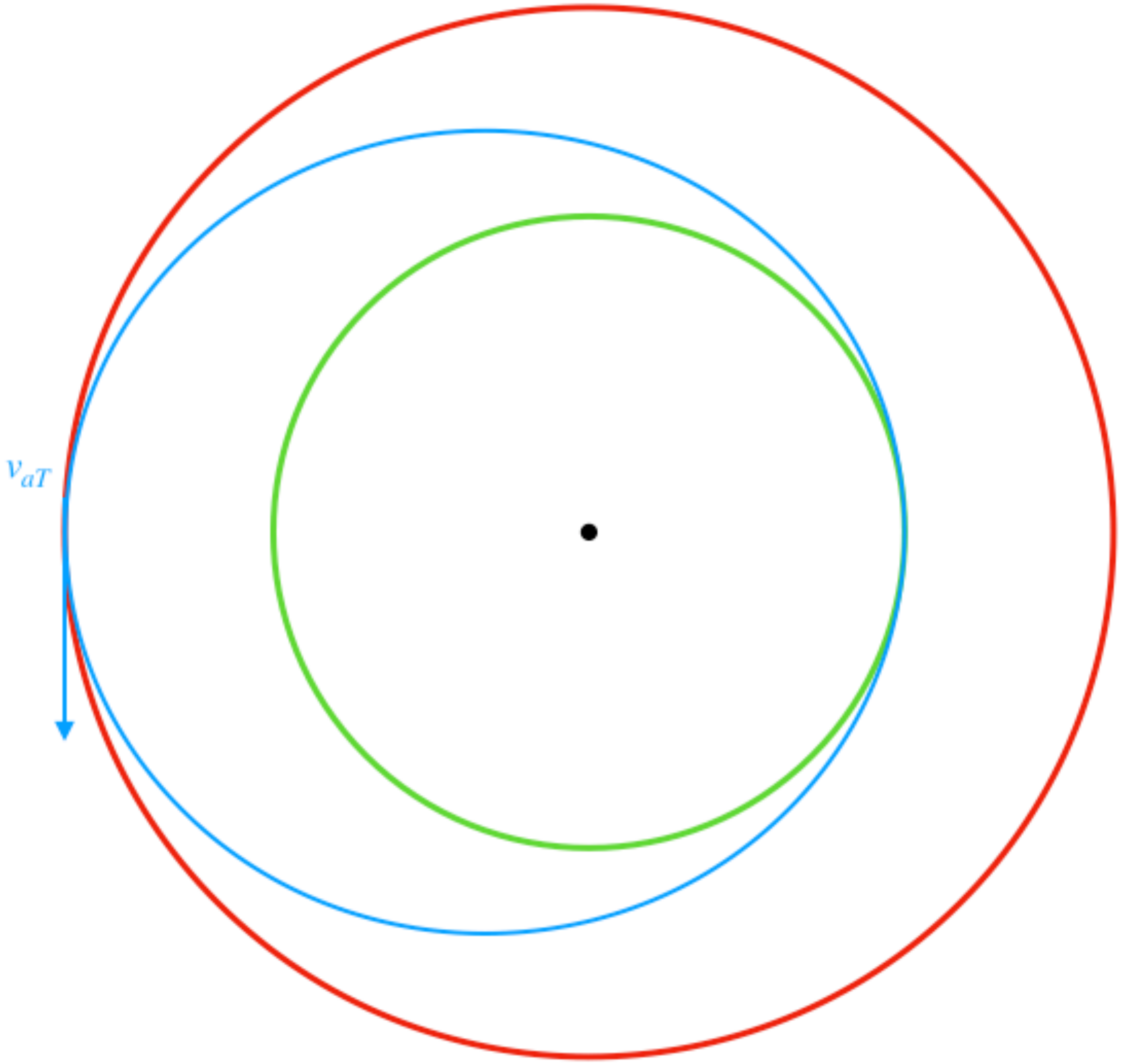

4. Calculate the Delta-V required for the first maneuver

$$\Delta V_1 = v_{pT} - v_{c1}$$

$$= \sqrt{\frac{GM}{r_1}} \sqrt{2\left(1 - \frac{1}{1 + \frac{r_2}{r_1}}\right)} - \sqrt{\frac{GM}{r_1}}$$

$$= \sqrt{\frac{GM}{r_1}} \left[ \sqrt{2\left(1 - \frac{1}{1 + \frac{r_2}{r_1}}\right)} - 1 \right]$$

# Hohmann transfer




5. Calculate the apogee velocity of the transfer orbit using vis-viva

$$v_{aT} = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

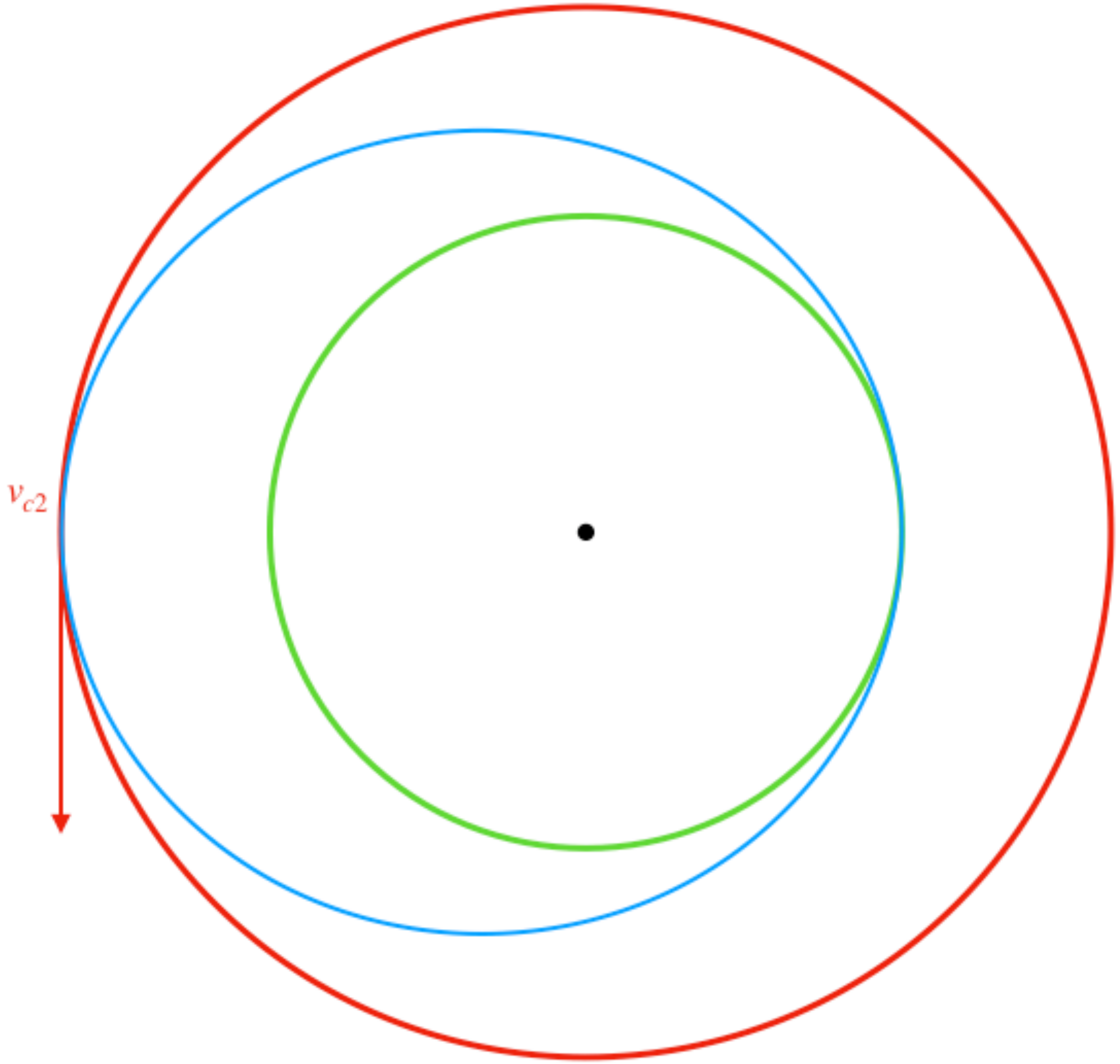
$$= \sqrt{GM\left(\frac{2}{r_2} - \frac{1}{a_T}\right)}$$

$$= \sqrt{GM\left(\frac{2}{r_2} - \frac{2}{r_1 + r_2}\right)}$$

$$= \sqrt{\frac{2GM}{r_2}\left(1 - \frac{1}{1 + \frac{r_1}{r_2}}\right)}$$

$$= \sqrt{\frac{GM}{r_2}}\sqrt{2\left(1 - \frac{1}{1 + \frac{r_1}{r_2}}\right)}$$

# Hohmann transfer



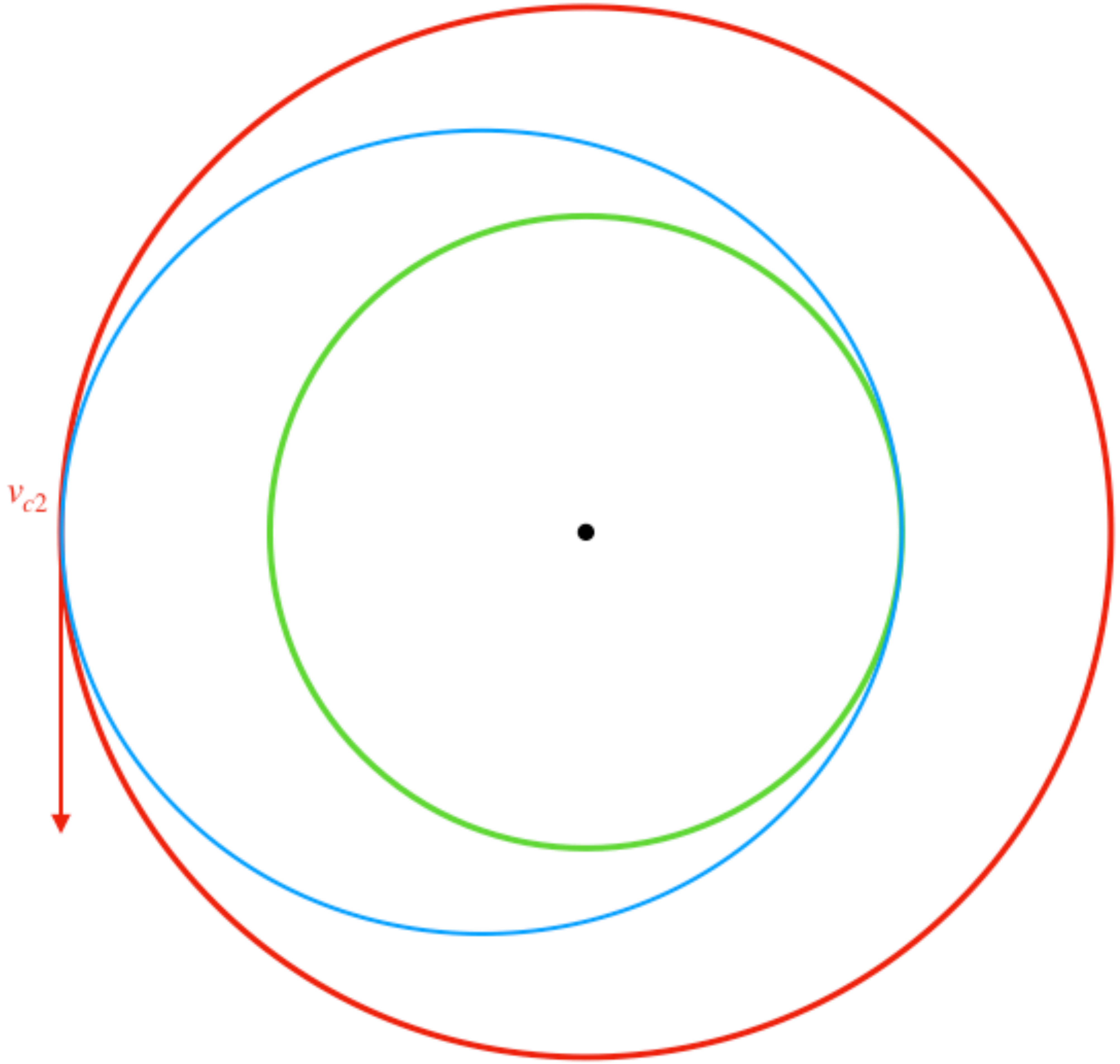

6. Calculate the circular velocity of the final orbit using vis-viva

$$v_{c2} = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$= \sqrt{GM\left(\frac{2}{r_2} - \frac{1}{r_2}\right)}$$

$$= \sqrt{\frac{GM}{r_2}}$$

# Hohmann transfer



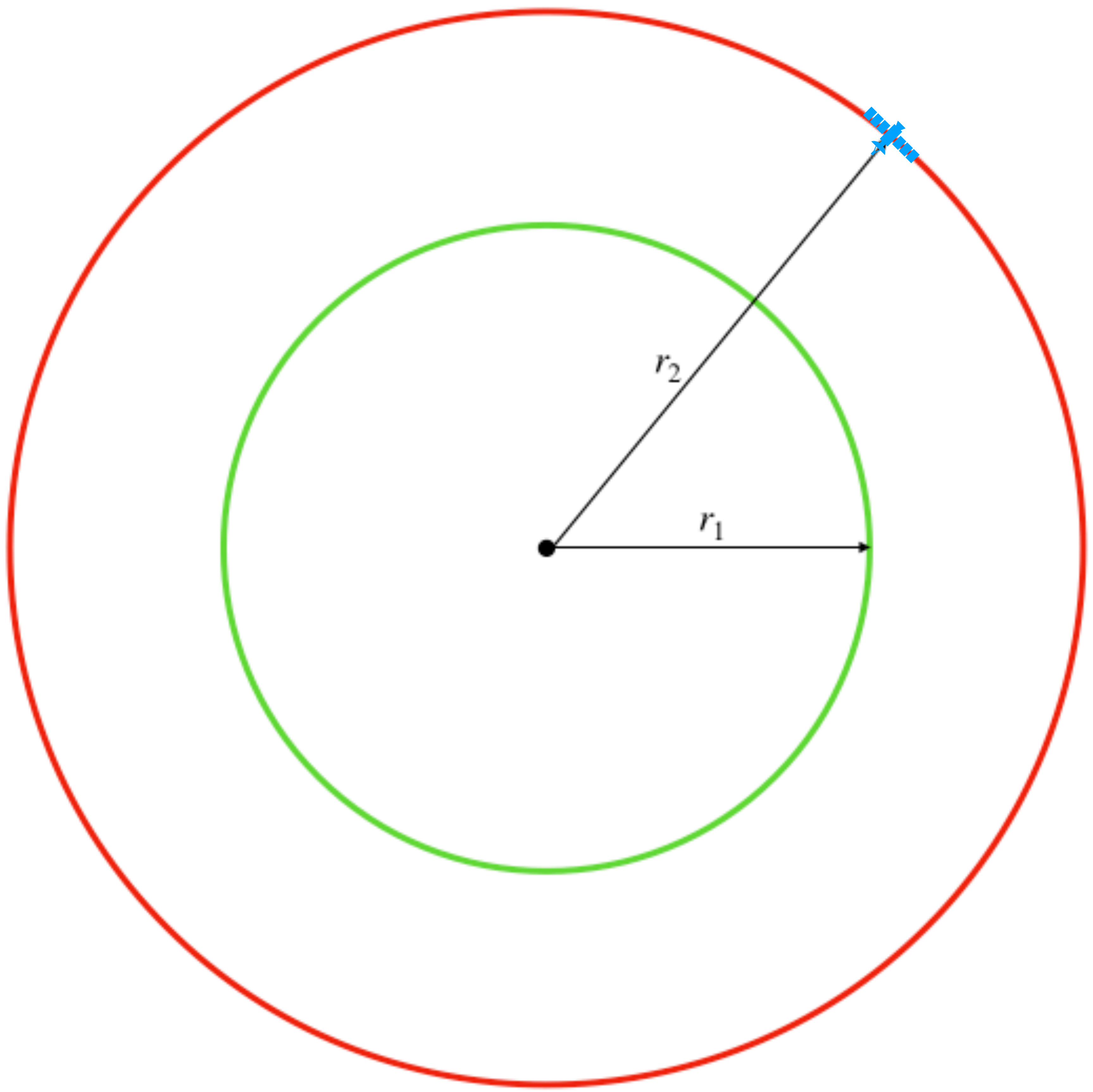

7. Calculate the Delta-V required for the second maneuver

$$\Delta V_2 = v_{c2} - v_{aT}$$

$$= \sqrt{\frac{GM}{r_2}} \sqrt{2\left(1 - \frac{1}{1 + \frac{r_1}{r_2}}\right)} - \sqrt{\frac{GM}{r_2}}$$

$$= \sqrt{\frac{GM}{r_2}} \left[\sqrt{2\left(1 - \frac{1}{1 + \frac{r_1}{r_2}}\right)} - 1\right]$$

# Hohmann transfer




8. Calculate the total required Delta-V

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2$$

$$= \sqrt{\frac{GM}{r_1}}\left[\sqrt{2\left(1 - \frac{1}{1 + \frac{r_2}{r_1}}\right)} - 1\right] + \sqrt{\frac{GM}{r_2}}\left[\sqrt{2\left(1 - \frac{1}{1 + \frac{r_1}{r_2}}\right)} - 1\right]$$

The vis-viva equation is useful for calculating:

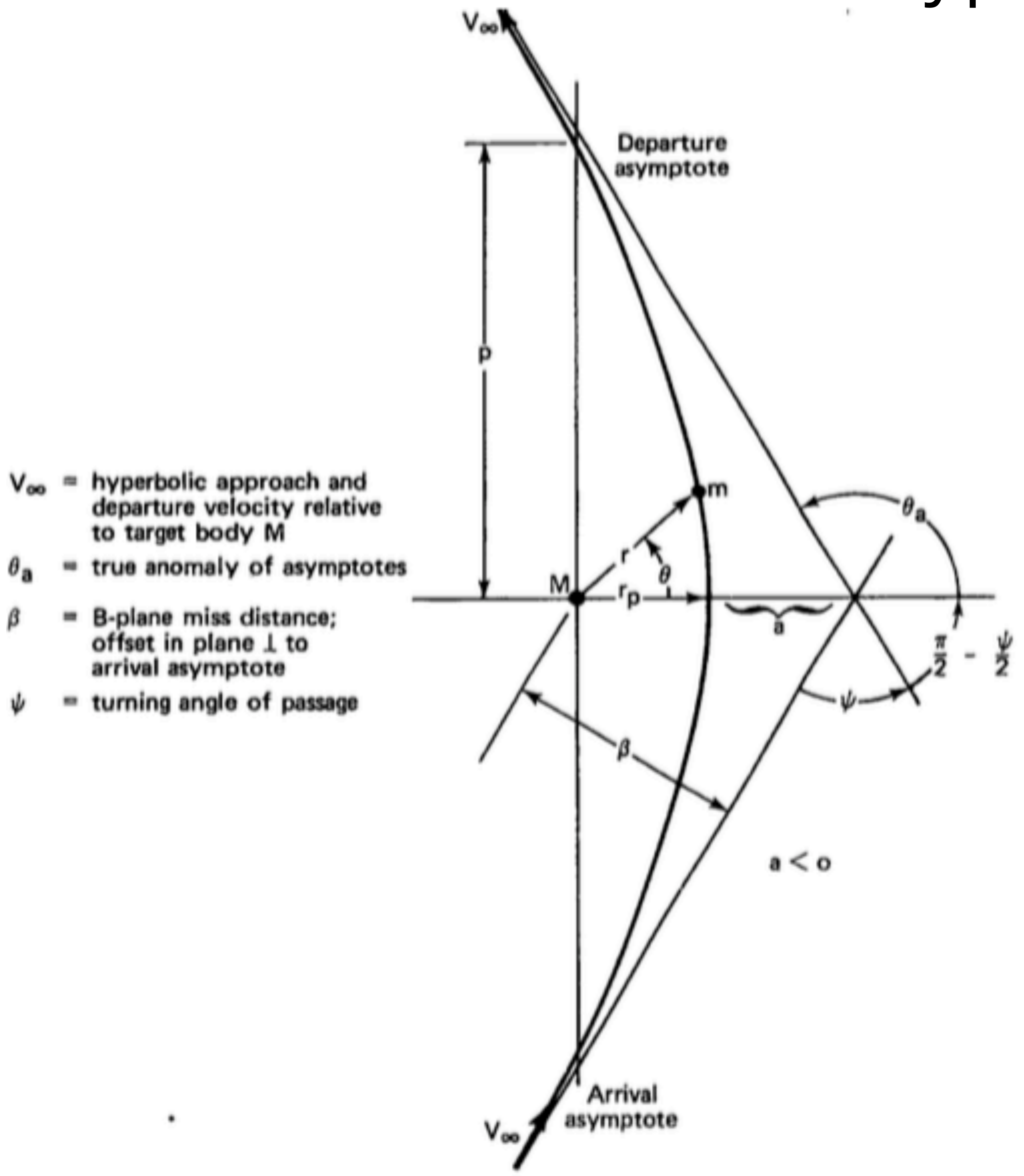
- Escape velocity
- Hohmann transfers
- Interplanetary Hohmann transfers

The vis-viva equation is useful for calculating:

- [illegible]
- Hohmann transfers
- Interplanetary Hohmann transfers

**Review of hyperbolic orbits**

# Hyperbolic orbits



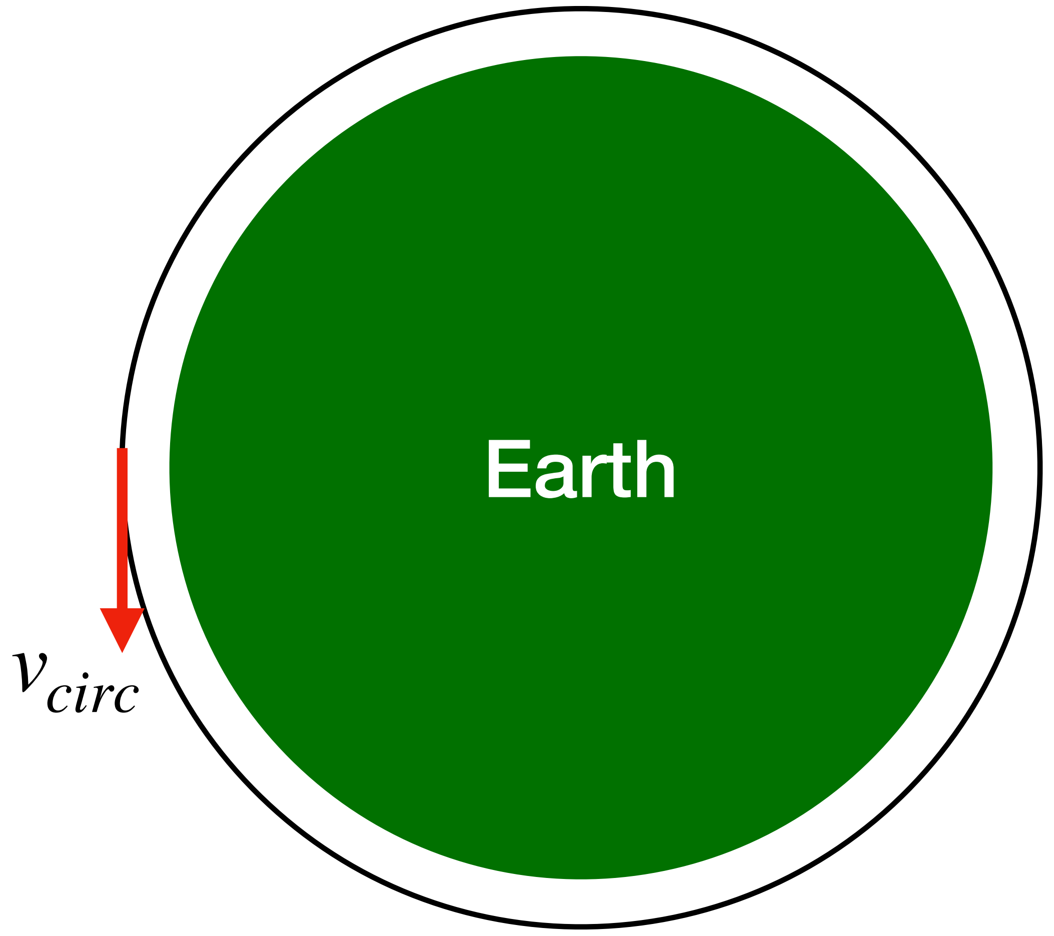

The vis-viva equation still holds!

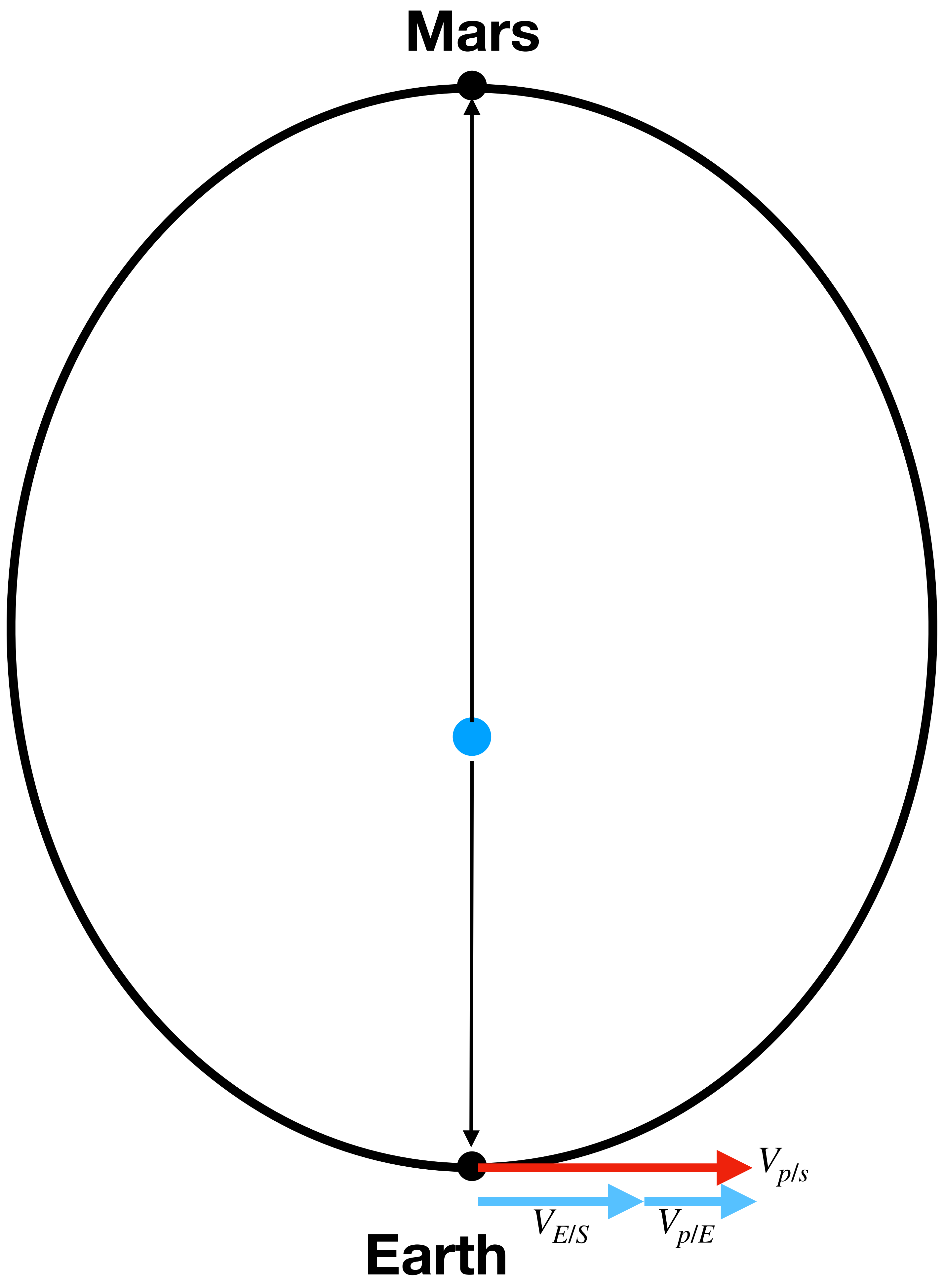
$$\frac{v^2}{2} - \frac{GM}{r} = -\frac{GM}{2a}$$

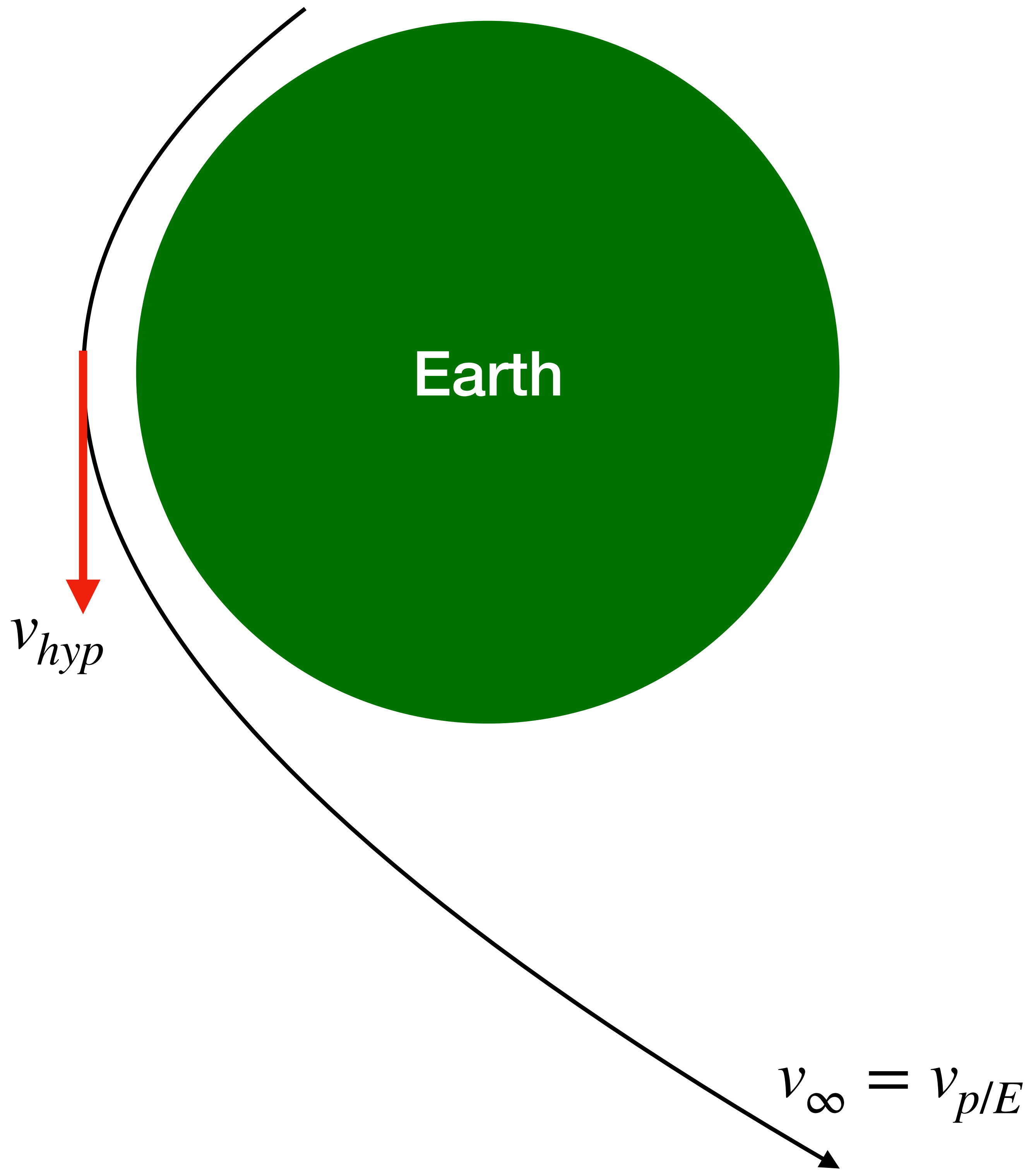
Unlike the escape velocity calculation, a spacecraft on a hyperbolic orbit about a planet leaves the sphere of influence of that planet with some *excess velocity*.

$$\frac{v_\infty^2}{2} = -\frac{GM}{2a}$$

# Interplanetary Hohmann Transfer



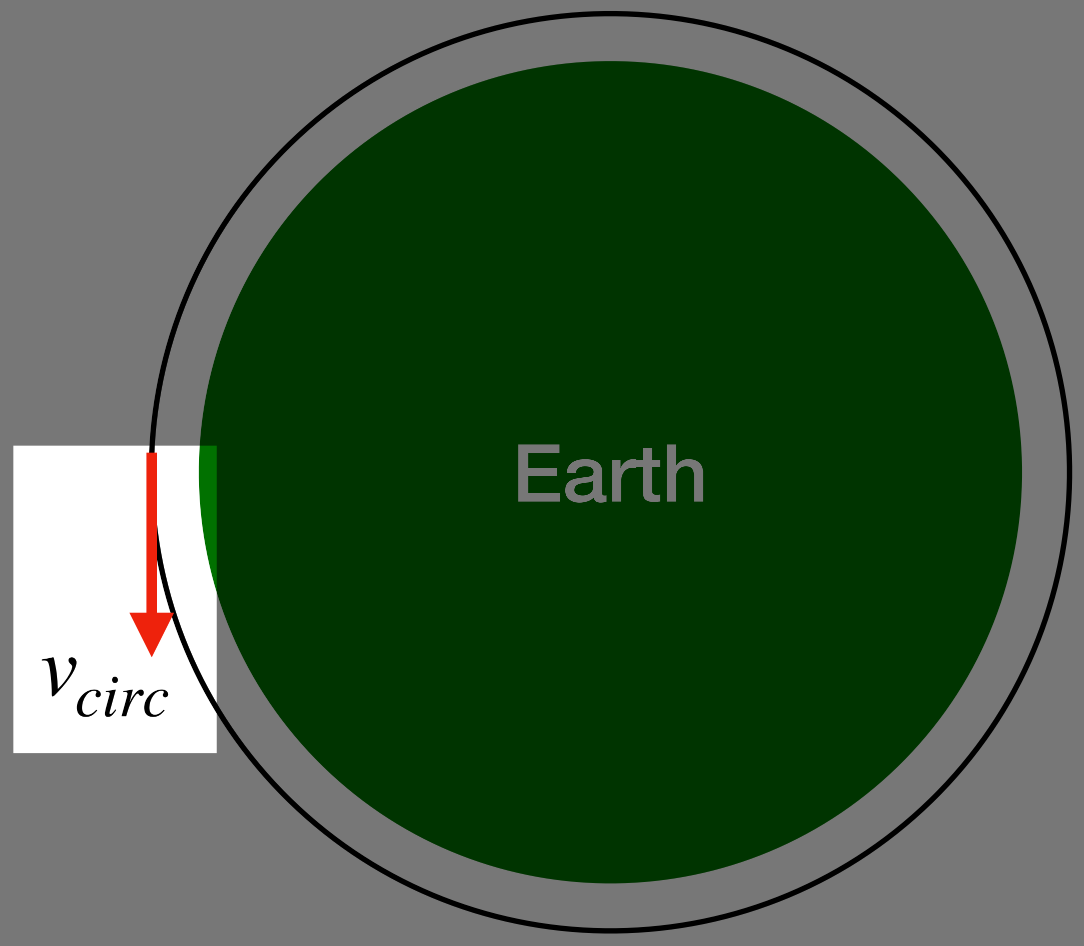



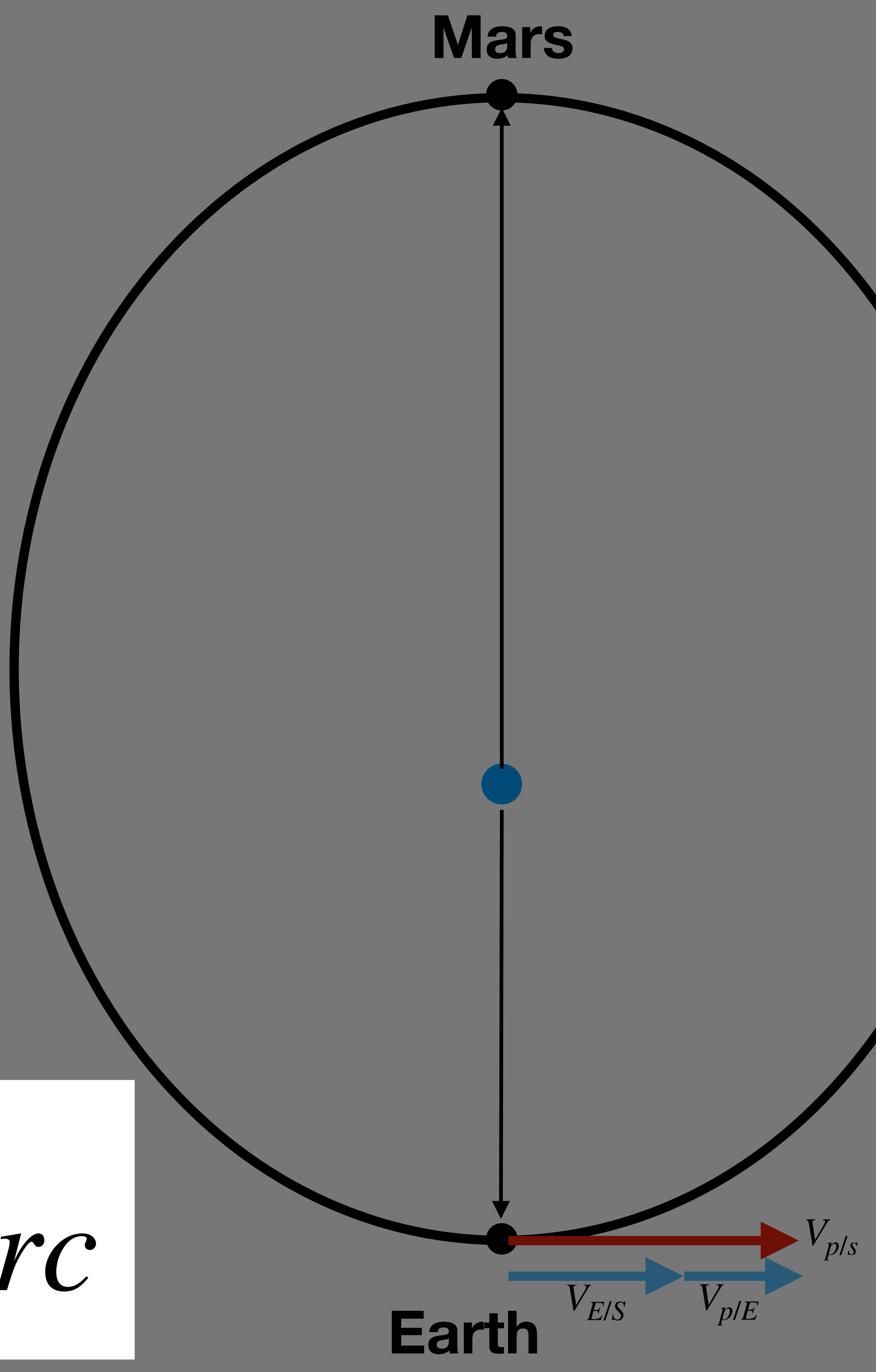

$$v_{circ}^2 = GM_E\left(\frac{2}{r} - \frac{1}{a}\right)$$

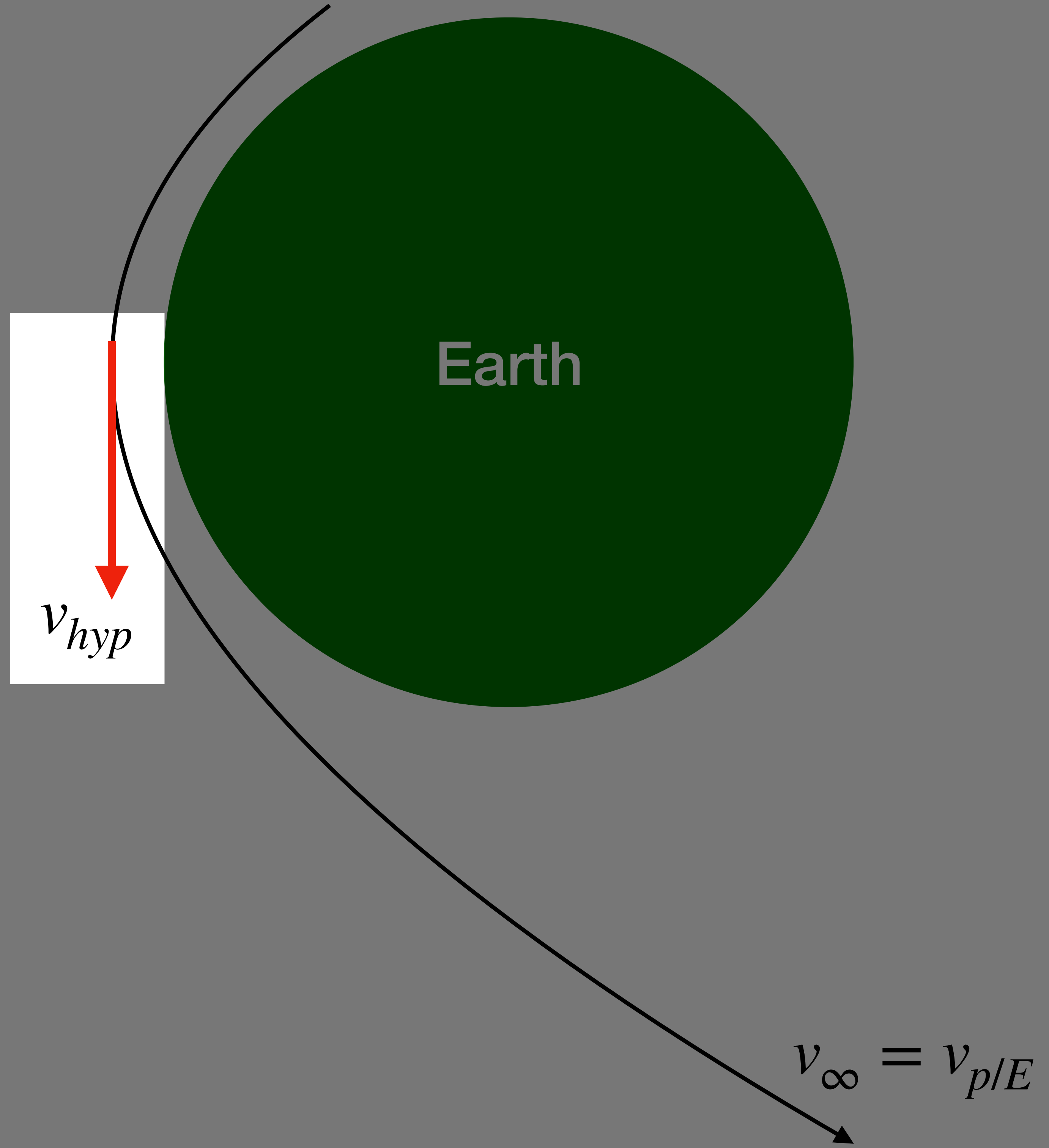
$$v_{p/S}^2 = GM_S\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$\frac{v_{hyp}^2}{2} - \frac{GM_E}{r} = -\frac{GM_E}{2a} = \frac{v_\infty^2}{2}$$

# Interplanetary Hohmann Transfer



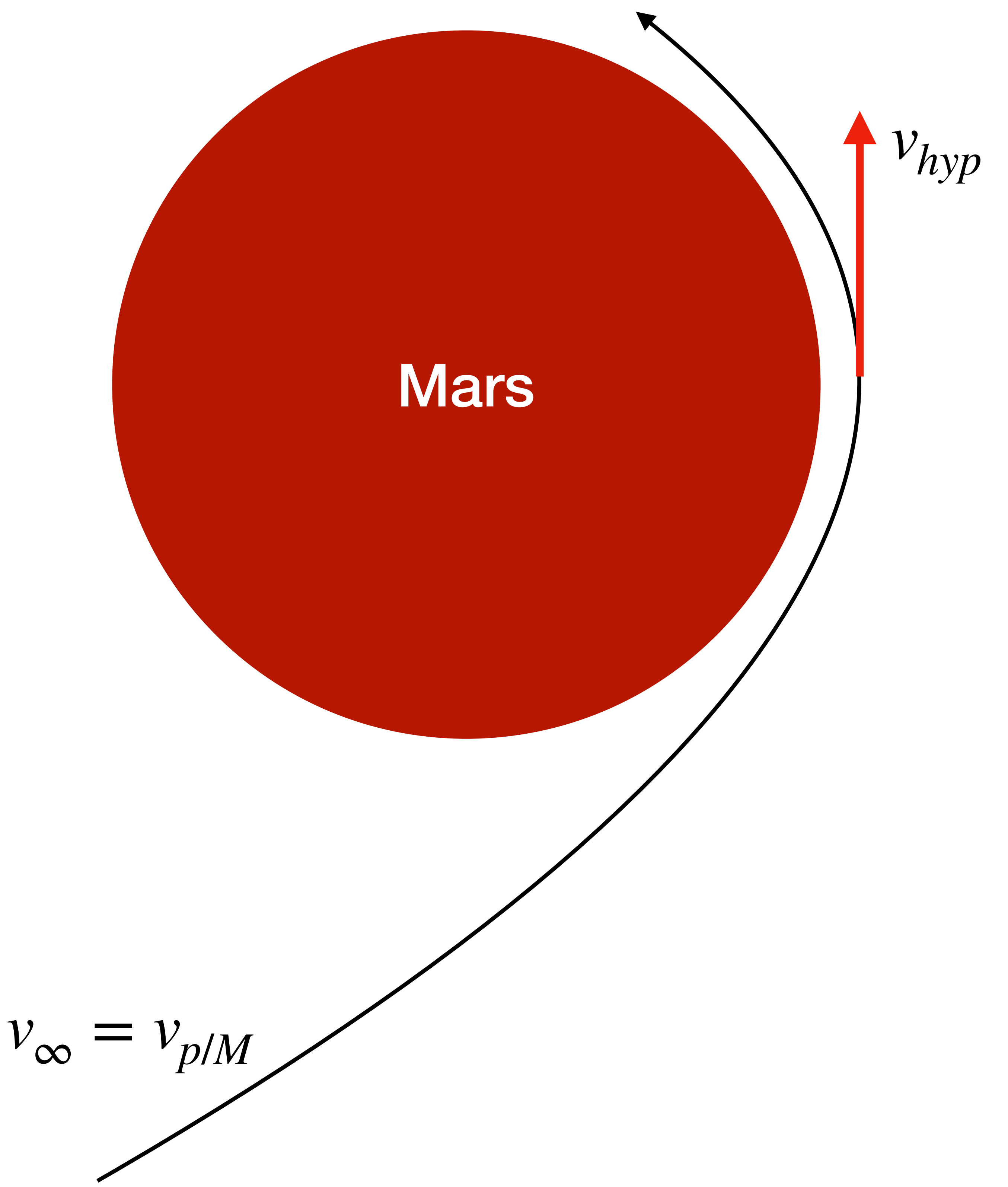

$$\Delta V_1 = v_{hyp} - v_{circ}$$

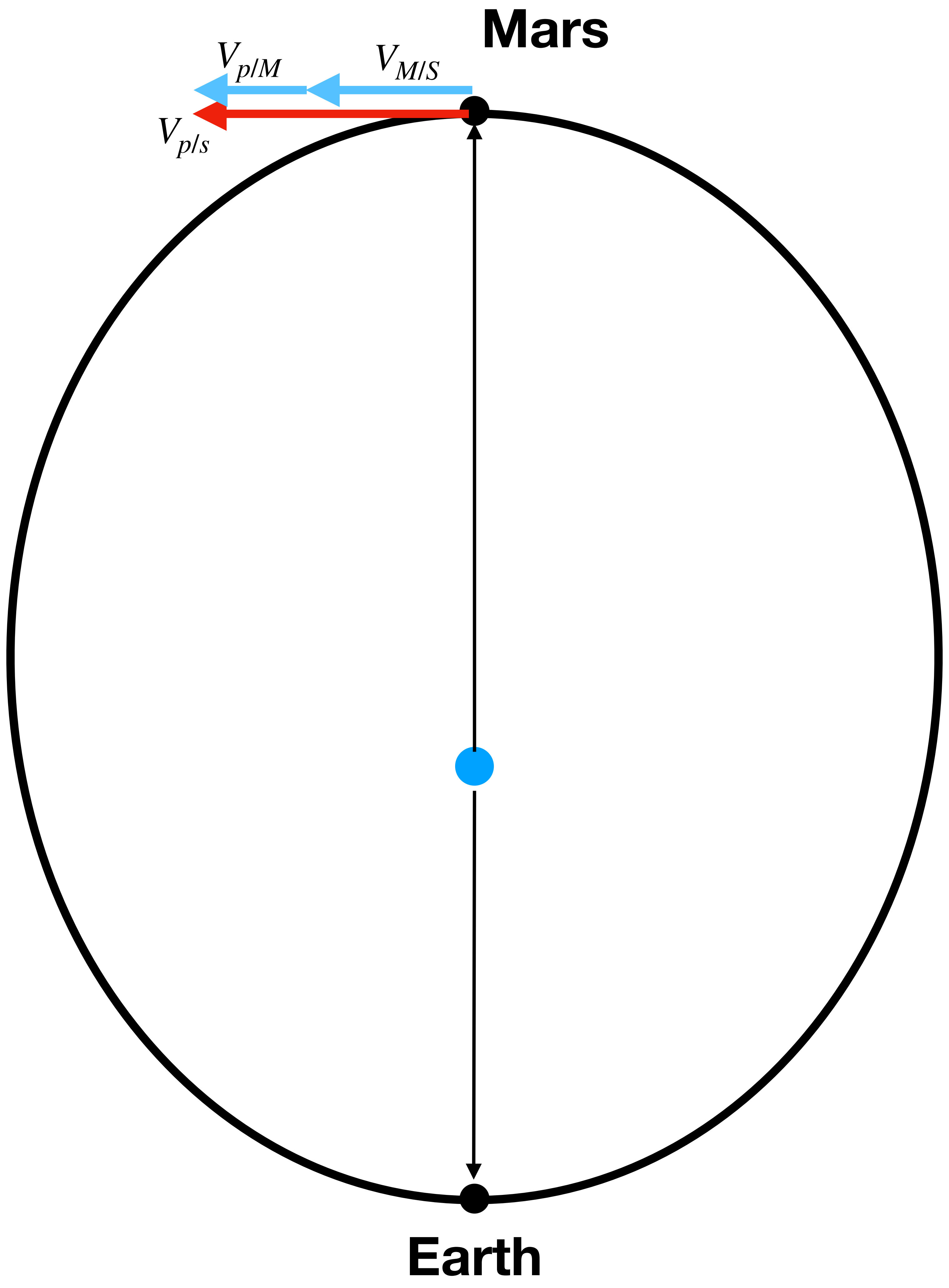
$$v_{circ}^2 = GM_E\left(\frac{2}{r} - \frac{1}{a}\right)$$

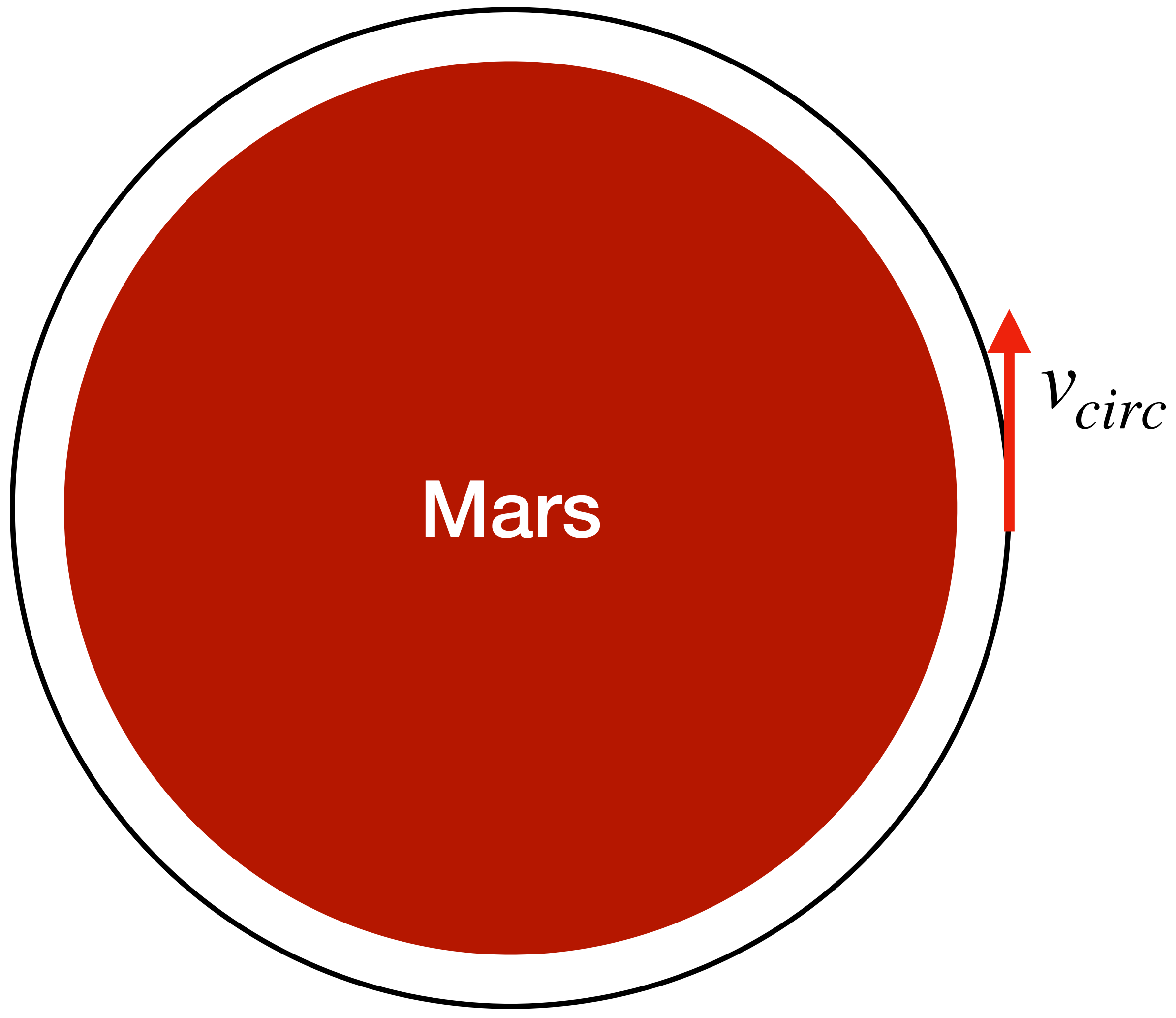
$$v_{p/S}^2 = GM_S\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$\frac{v_{hyp}^2}{2} - \frac{GM_E}{r} = -\frac{GM_E}{2a} = \frac{v_\infty^2}{2}$$

# Interplanetary Hohmann Transfer



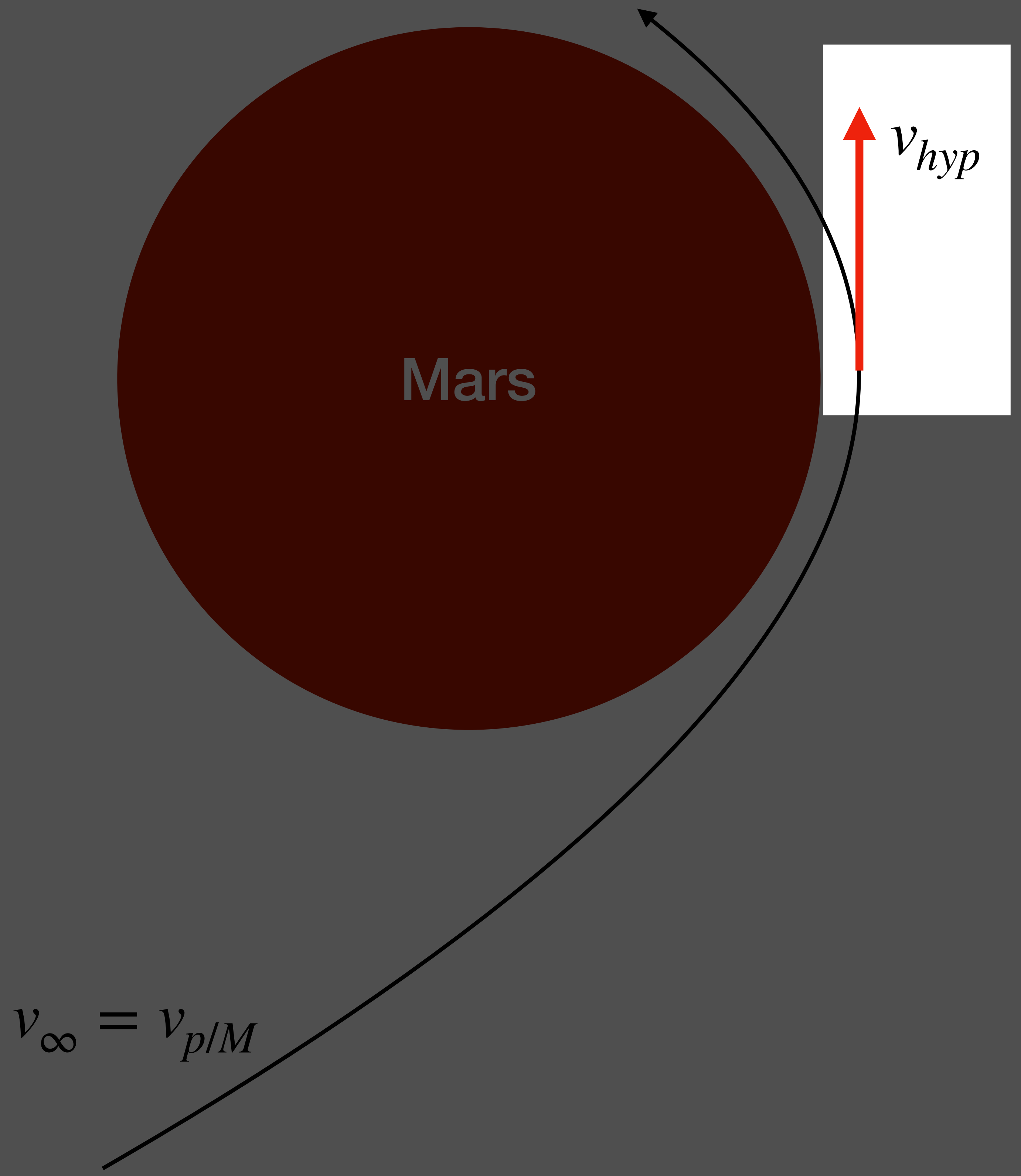



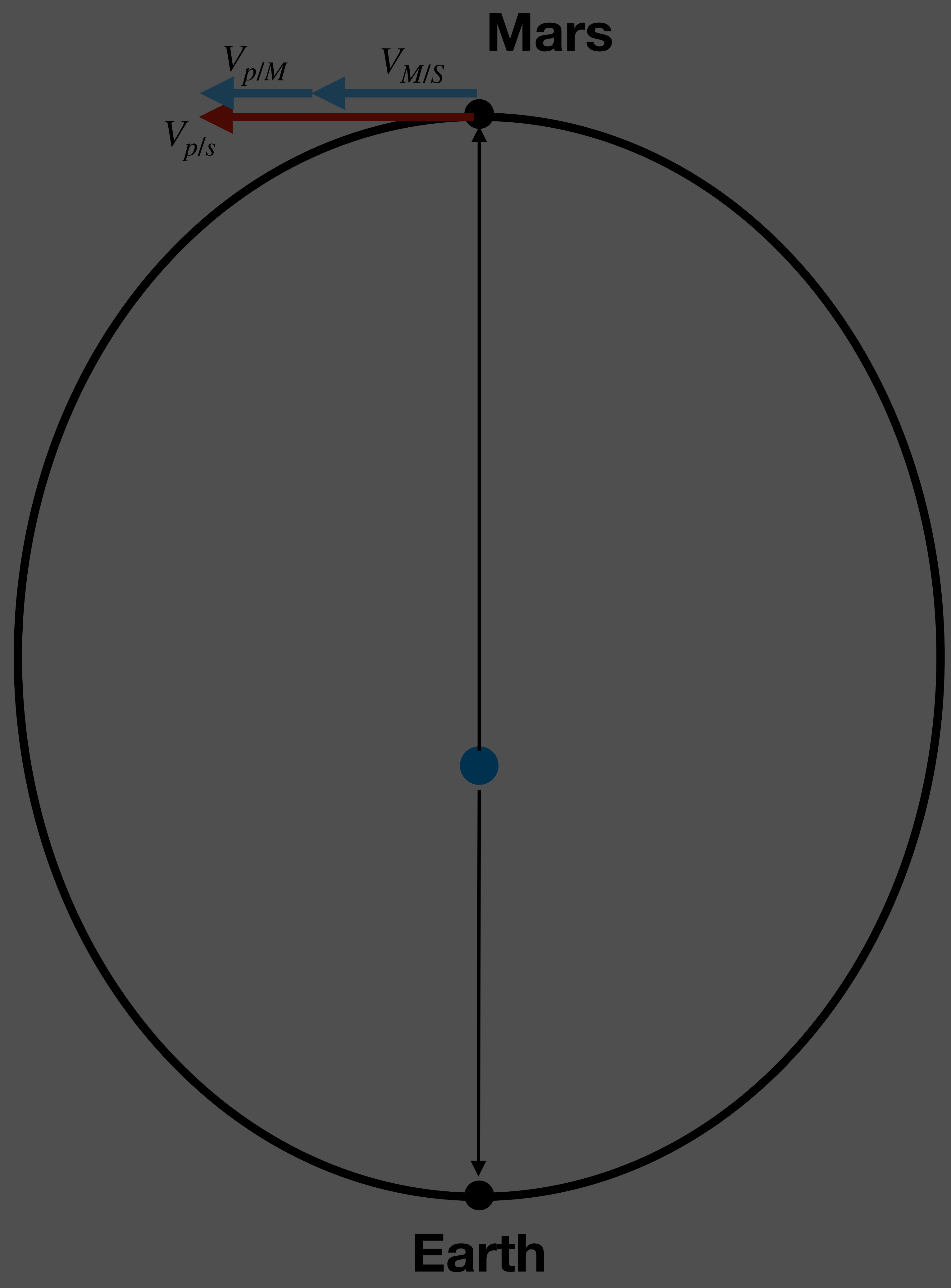

$$\frac{v_{hyp}^2}{2} - \frac{GM_M}{r} = -\frac{GM_M}{2a} = \frac{v_\infty^2}{2}$$

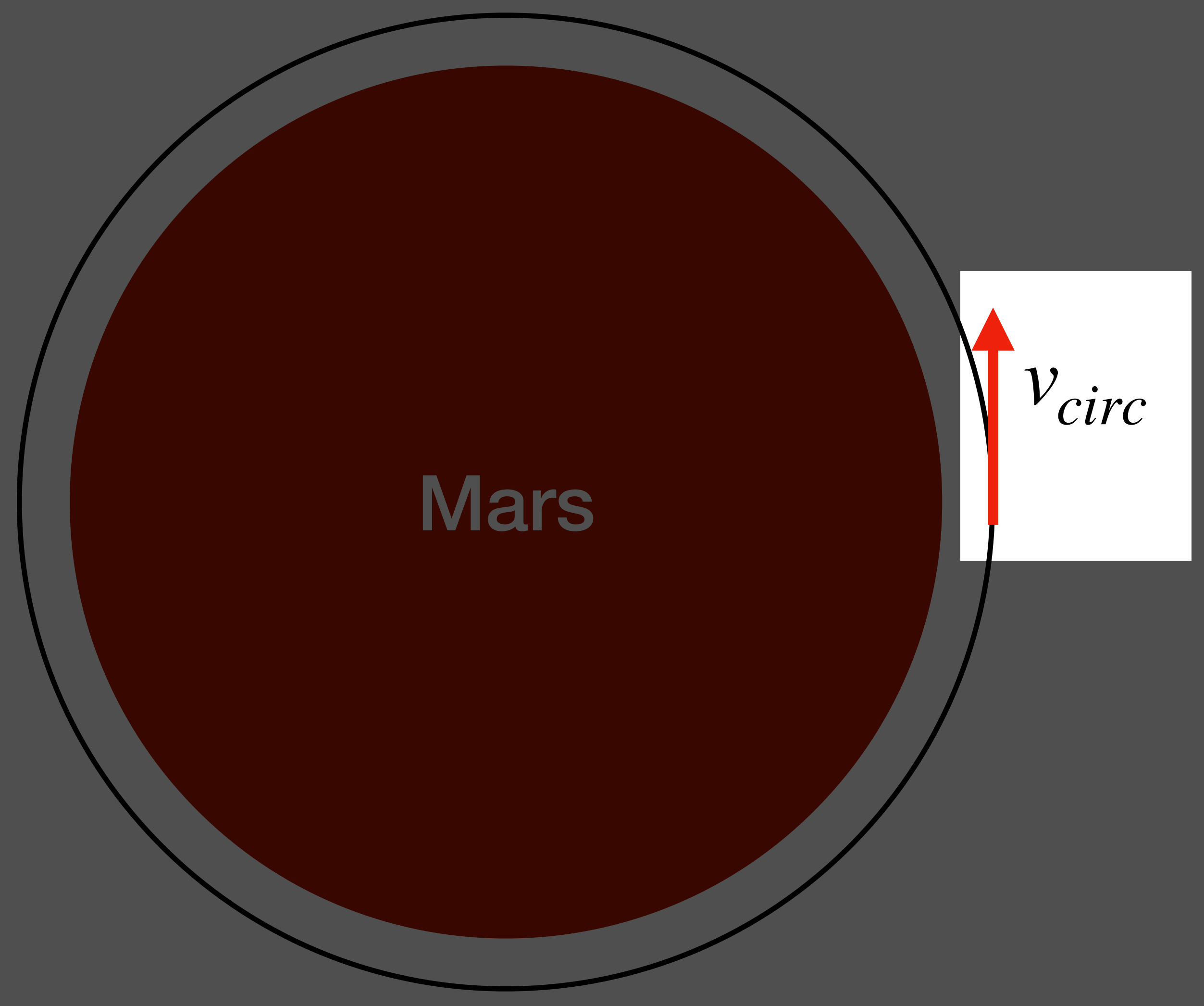
$$v_{p/S}^2 = GM_S\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$v_{circ}^2 = GM_M\left(\frac{2}{r} - \frac{1}{a}\right)$$

# Interplanetary Hohmann Transfer



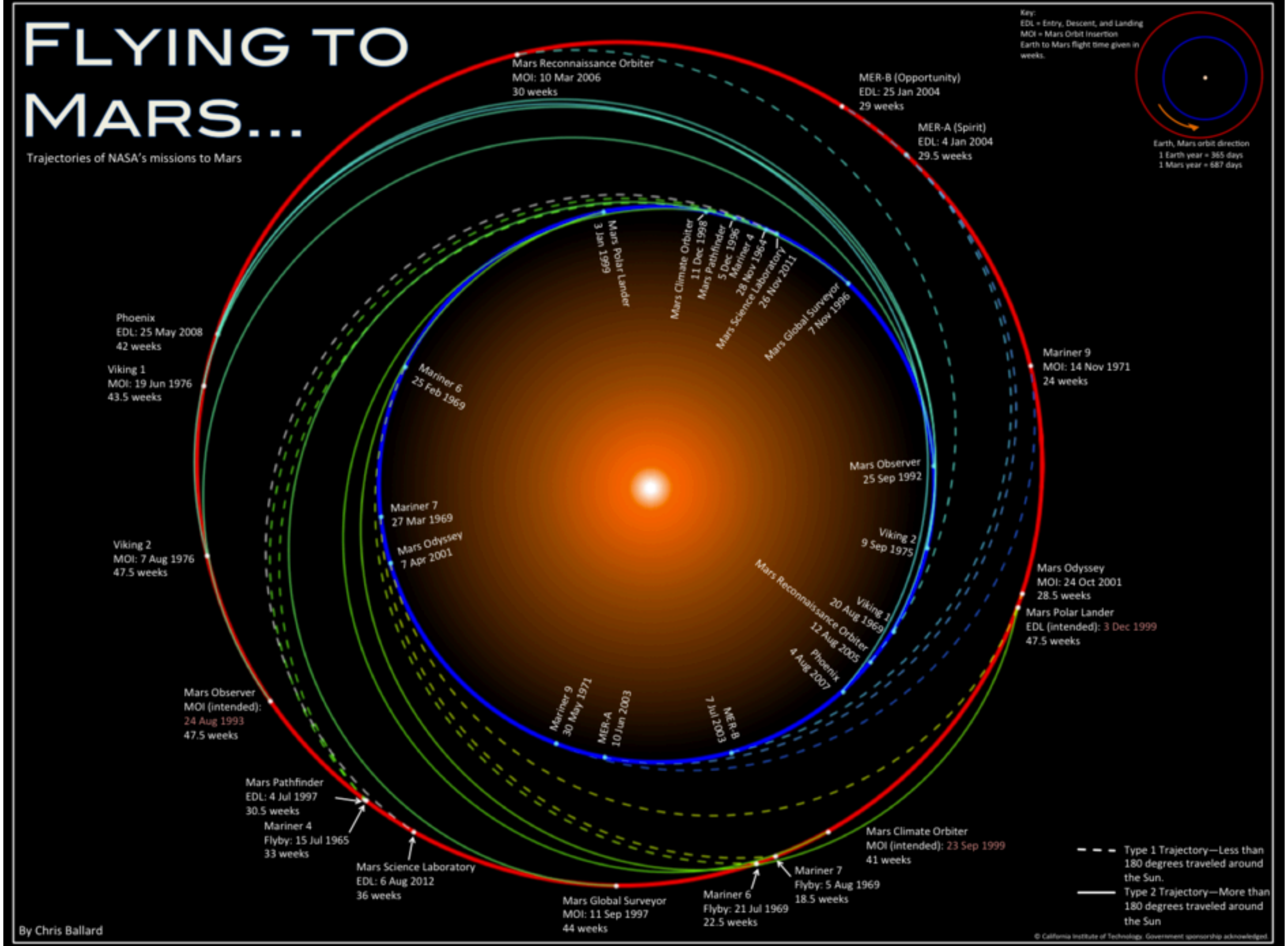




$$\Delta V_2 = v_{hyp} - v_{circ}$$

$$\frac{v_{hyp}^2}{2} - \frac{GM_M}{r} = -\frac{GM_M}{2a} = \frac{v_\infty^2}{2}$$

$$v_{p/S}^2 = GM_S\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$v_{circ}^2 = GM_M\left(\frac{2}{r} - \frac{1}{a}\right)$$



Flying to Mars...
Trajectories of NASA's missions to Mars
Key:
EDL = Entry, Descent, and Landing
MOI = Mars Orbit Insertion
Earth to Mars flight time given in weeks.
Earth, Mars orbit direction
1 Earth year = 365 days
1 Mars year = 687 days
Mars Reconnaissance Orbiter
MOI: 10 Mar 2006
30 weeks
MER-B (Opportunity)
EDL: 25 Jan 2004
29 weeks
MER-A (Spirit)
EDL: 4 Jan 2004
29.5 weeks
Phoenix
EDL: 25 May 2008
42 weeks
Viking 1
MOI: 19 Jun 1976
43.5 weeks
Mariner 9
MOI: 14 Nov 1971
24 weeks
Viking 2
MOI: 7 Aug 1976
47.5 weeks
Mars Odyssey
MOI: 24 Oct 2001
28.5 weeks
Mars Polar Lander
EDL (intended): 3 Dec 1999
47.5 weeks
Mars Observer
MOI (intended):
24 Aug 1993
47.5 weeks
Mars Pathfinder
EDL: 4 Jul 1997
30.5 weeks
Mariner 4
Flyby: 15 Jul 1965
33 weeks
Mars Science Laboratory
EDL: 6 Aug 2012
36 weeks
Mars Global Surveyor
MOI: 11 Sep 1997
44 weeks
Mariner 6
Flyby: 21 Jul 1969
22.5 weeks
Mariner 7
Flyby: 5 Aug 1969
18.5 weeks
Mars Climate Orbiter
MOI (intended): 23 Sep 1999
41 weeks
Mars Polar Lander 3 Jan 1999
Mars Climate Orbiter 11 Dec 1998
Mars Pathfinder 5 Dec 1996
Mariner 4 28 Nov 1964
Mars Science Laboratory 26 Nov 2011
Mars Global Surveyor 7 Nov 1996
Mariner 6 25 Feb 1969
Mars Observer 25 Sep 1992
Mariner 7 27 Mar 1969
Mars Odyssey 7 Apr 2001
Viking 2 9 Sep 1975
Viking 1 20 Aug 1975
Mars Reconnaissance Orbiter 12 Aug 2005
Phoenix 4 Aug 2007
Mariner 9 30 May 1971
MER-A 10 Jun 2003
MER-B 7 Jul 2003
Type 1 Trajectory—Less than 180 degrees traveled around the Sun.
Type 2 Trajectory—More than 180 degrees traveled around the Sun
By Chris Ballard
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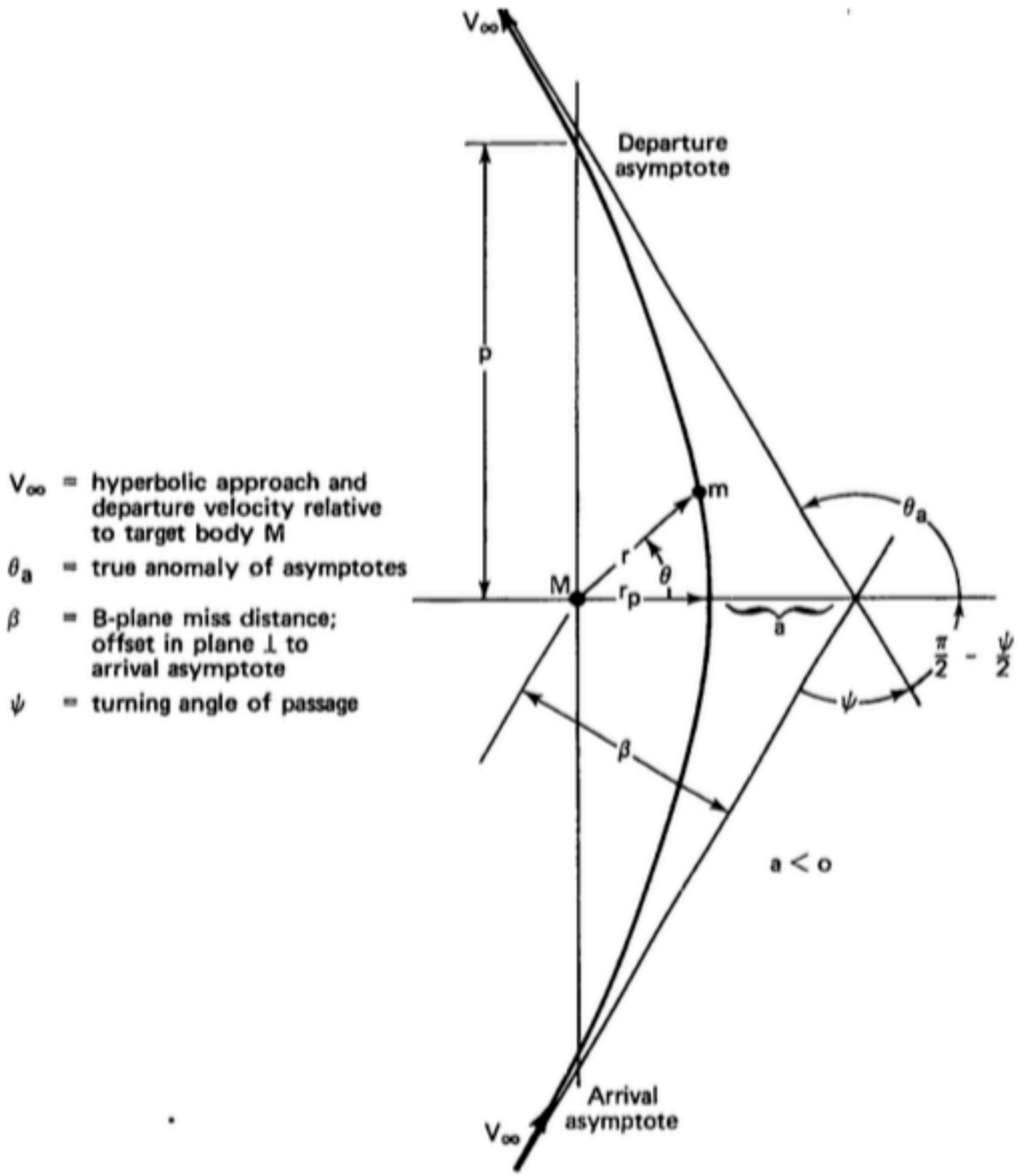
If we don’t execute a capture burn, we can perform a *flyby*.

# Interplanetary assists - flyby maneuvers



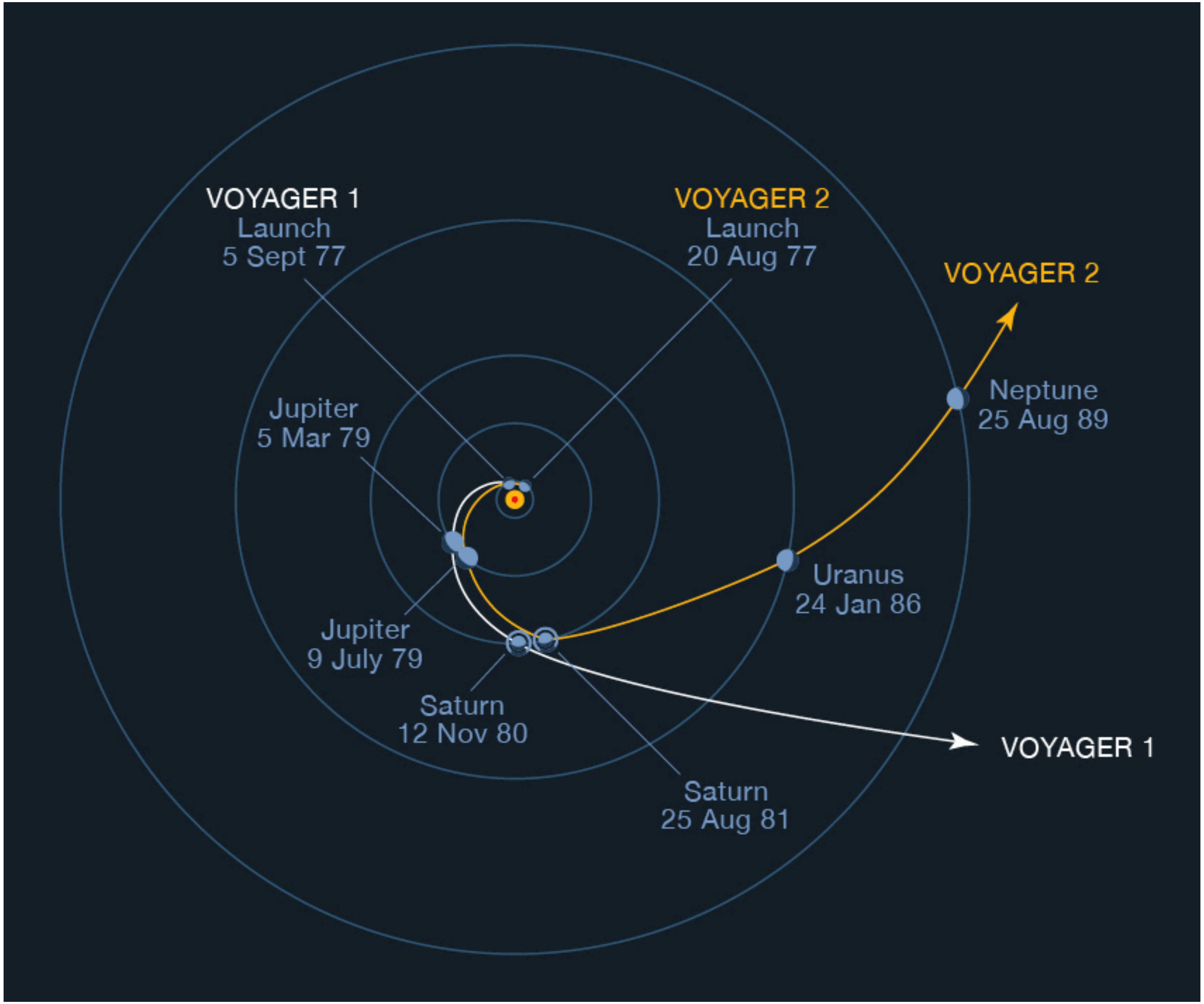

The spacecraft approaches the planet with a speed (with respect to the planet) of $v_\infty$

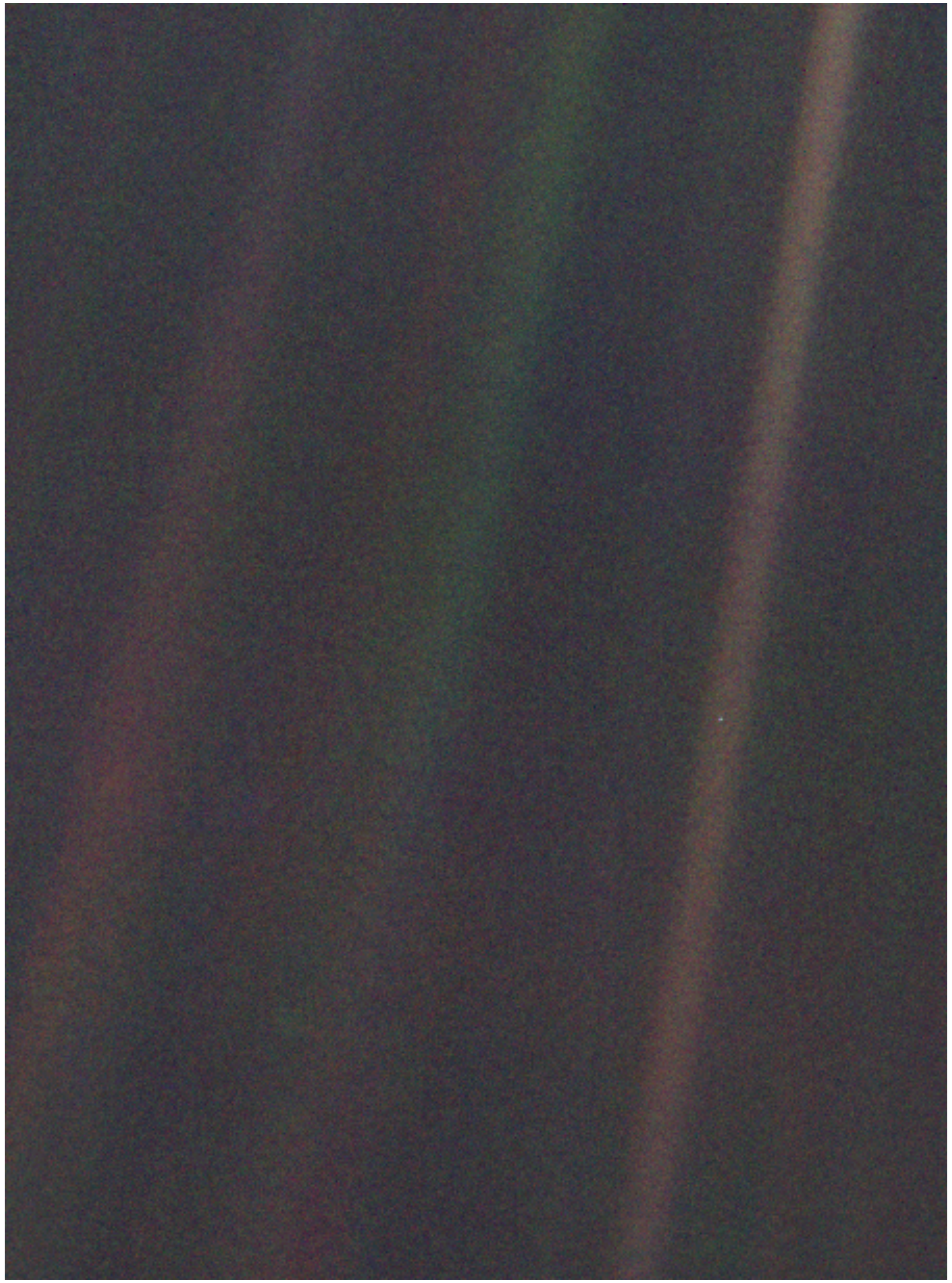
It leaves the planet with the *same speed with respect to the planet*, but at a *different angle*.

$$\sin\frac{\Psi}{2} = \frac{1}{e} = \left(\frac{1 + v_\infty^2 r_p}{GM}\right)^{-1}$$

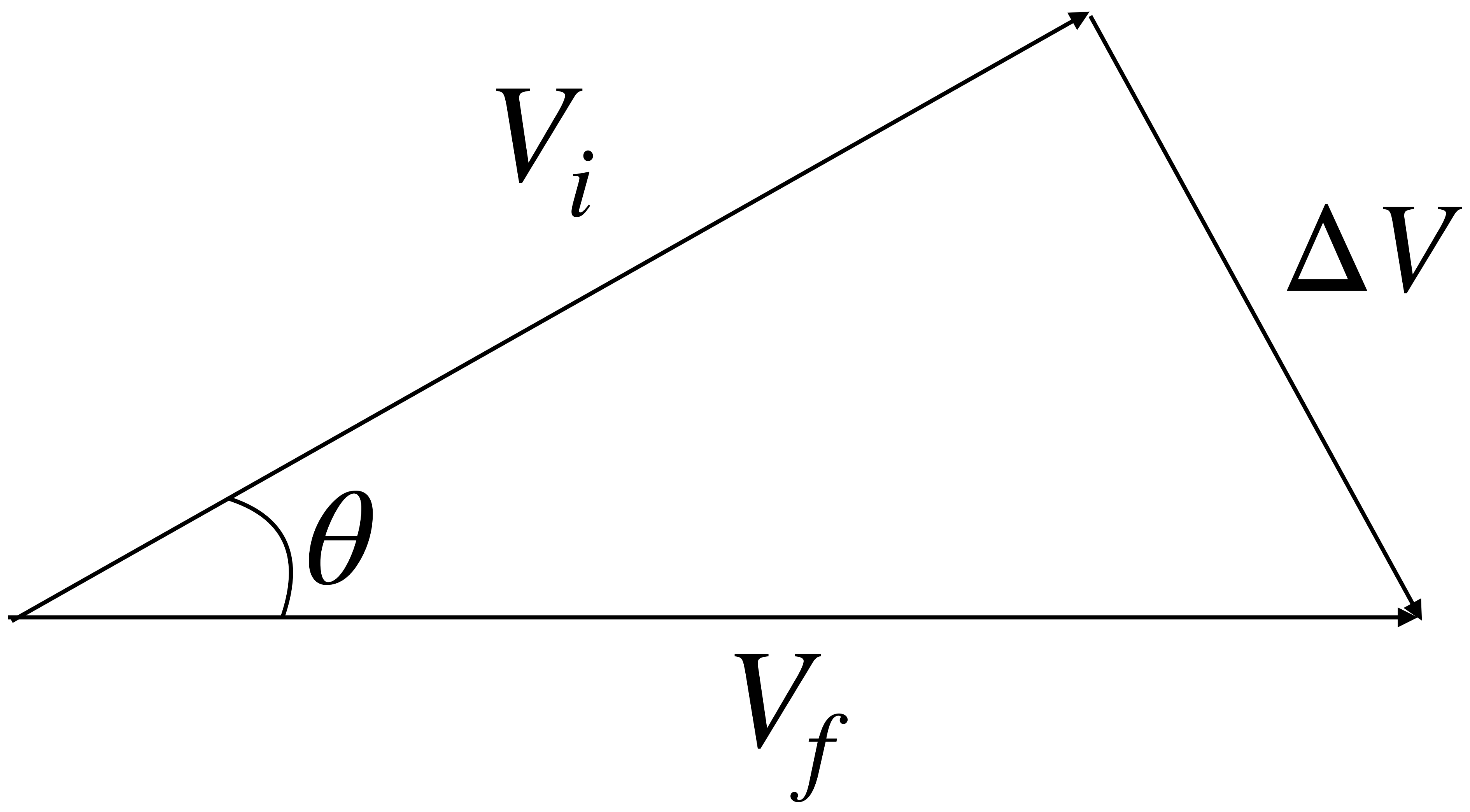
This rotation creates a Delta-V with respect to the Sun.

$$\Delta V = 2V_\infty \sin\left(\frac{\Psi}{2}\right)$$


VOYAGER 1
Launch
5 Sept 77
VOYAGER 2
Launch
20 Aug 77
VOYAGER 2
Jupiter
5 Mar 79
Neptune
25 Aug 89
Uranus
24 Jan 86
Jupiter
9 July 79
Saturn
12 Nov 80
VOYAGER 1
Saturn
25 Aug 81



# Out-of-plane maneuvers




Consider a simple plane change

$$\theta = \Delta i$$

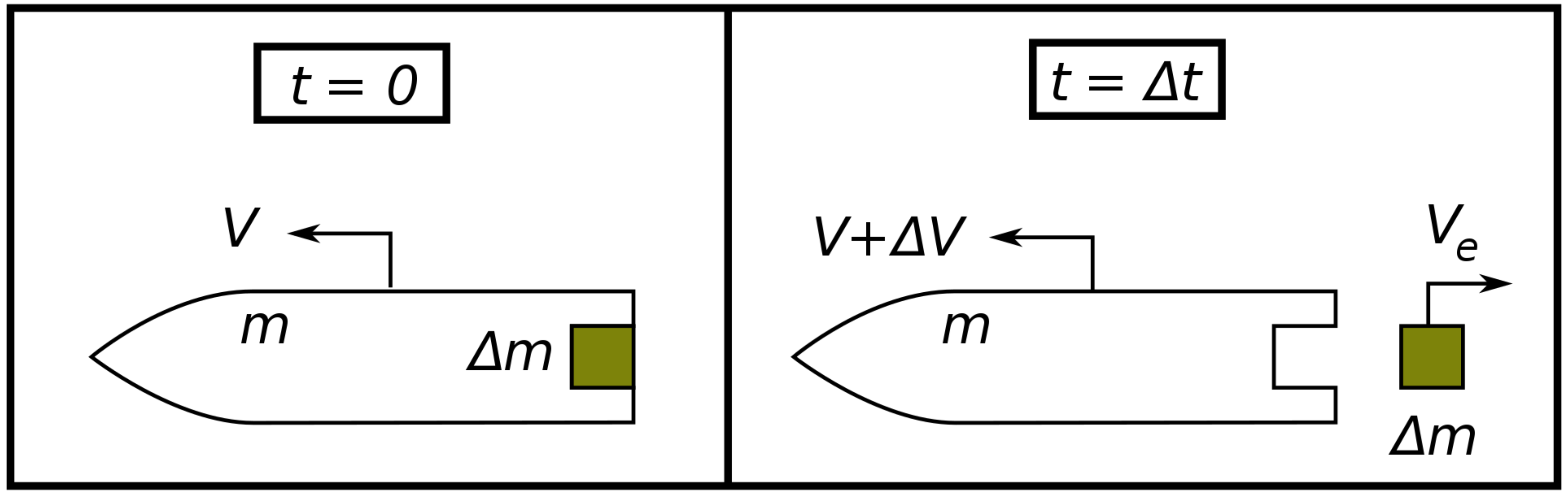
$$\Delta V = 2V_i \sin\left(\frac{\Delta i}{2}\right)$$

The change in velocity is proportional to the initial velocity. It costs a lot of propellant to change the inclination of an orbit.

Best to let physics help you with these maneuvers.

We execute these Delta-V maneuvers with *propulsion*.

# The rocket equation




**Derived in the lecture supplements.**

$$\Delta V = v_e \ln\left(\frac{m_{prop} + m_{dry}}{m_{dry}}\right)$$

$$m_{prop} = m_{dry}\left(e^{\frac{\Delta V}{v_e}} - 1\right)$$

$$\boxed{v_e = g_0 \cdot ISP}$$

# The rocket equation

Some things to note:

- For a given ΔV, propellant mass increases linearly with dry mass.
- There is an “exponential wall” associated with ΔV. Mass ratio increases exponentially as ΔV increases.
- For a given propellant mass and dry mass, ΔV increases linearly with ISP

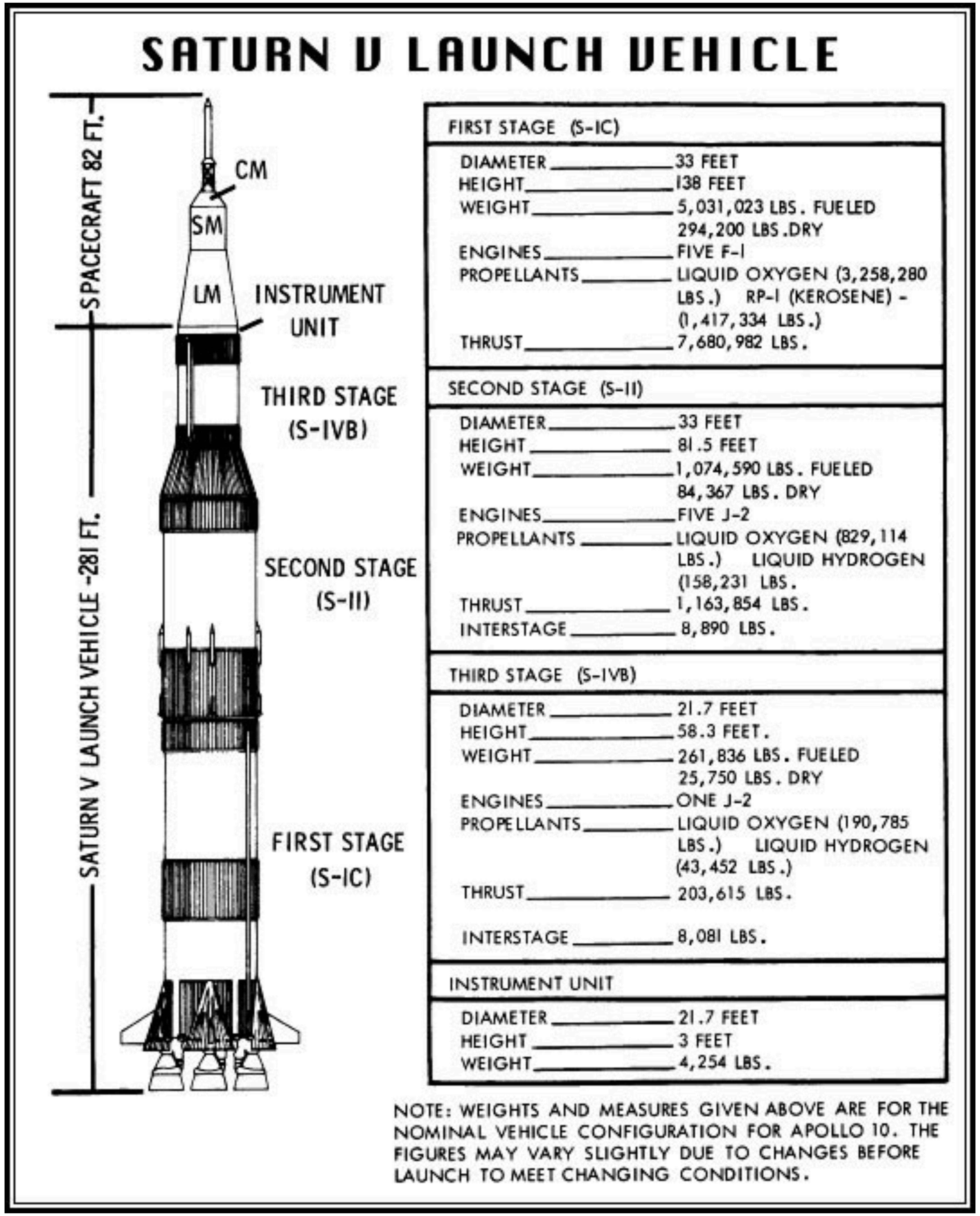
$$\Delta V = v_e \ln\left(\frac{m_{prop} + m_{dry}}{m_{dry}}\right)$$

$$m_{prop} = m_{dry}\left(e^{\frac{\Delta V}{v_e}} - 1\right)$$

$$\boxed{v_e = g_0 \cdot ISP}$$

# Staging

- To date, there are no single-stage to orbit rockets
- Staging is used to jettison the dry mass of expended stages
- The rocket equation is applied to each stage, taking into account that each stage must accelerate subsequent stages.



# Chemical Propulsion

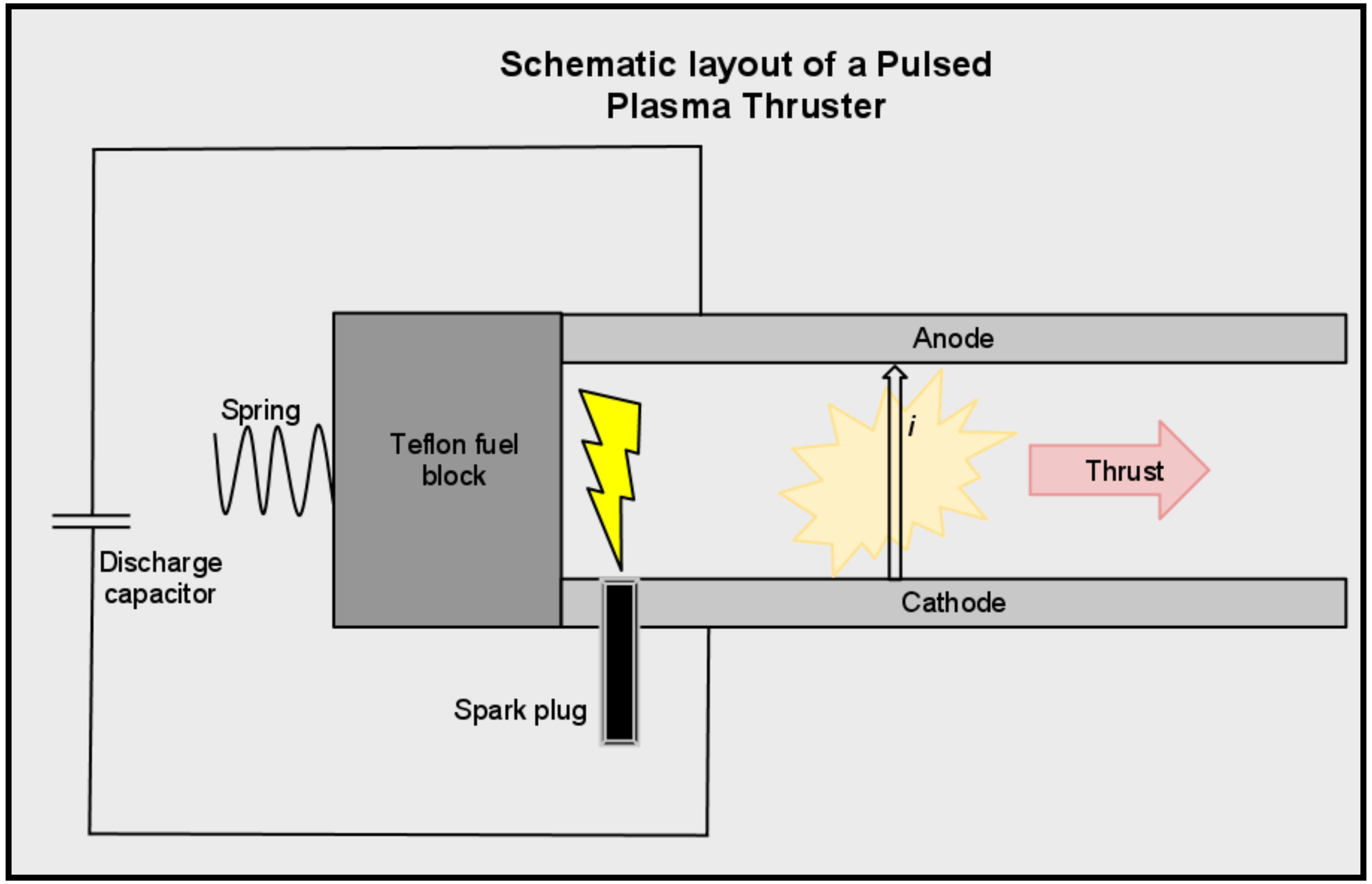
- Energy is stored in the molecular bonds of the propellant, and is transformed into kinetic energy via expansion
- Includes **cold gas thrusters** (ISP~75 sec), **liquid propellants** (ISP~400 sec, ΔV>1 km/s), and **solid propellants** (ISP~200s)

# Electric Propulsion

- Energy comes from accelerating particles through magnetic fields
- Very high ISP (up to ~10,000 sec), but low thrust (<1N)
- Include electrostatic and electromagnetic thrusters

$$F = qv \times B$$

$$F = qE$$




**Pulsed plasma thruster**

| Thruster | Specific Impulse (s) | Input Power (kW) | Efficiency Range (%) | Propellant |
|---|---|---|---|---|
| Cold gas | 50–75 | — | — | Various |
| Chemical (monopropellant) | 150–225 | — | — | $N_2H_4$<br>$H_2O_2$ |
| Chemical (bipropellant) | 300–450 | — | — | Various |
| Resistojet | 300 | 0.5–1 | 65–90 | $N_2H_4$ monoprop |
| Arcjet | 500–600 | 0.9–2.2 | 25–45 | $N_2H_4$ monoprop |
| Ion thruster | 2500–3600 | 0.4–4.3 | 40–80 | Xenon |
| Hall thrusters | 1500–2000 | 1.5–4.5 | 35–60 | Xenon |
| PPTs | 850–1200 | <0.2 | 7–13 | Teflon |

# Other propulsion

- Solar sails
- Tethers
- Gigawatt lasers (?)